Swiss National Science Foundation

Université de Genève

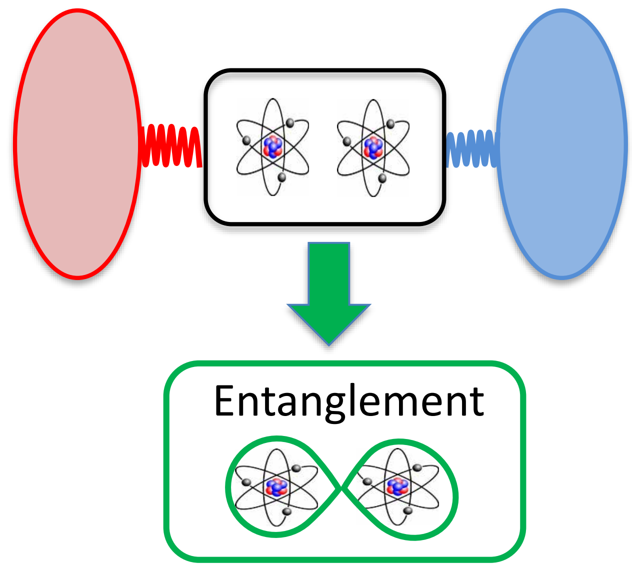
Quantum thermal machines

(Quantum dynamics, Quantum transport, Quantum information)

Géraldine Haack

FNS Prima grant

Ligne Blonay - Chamby

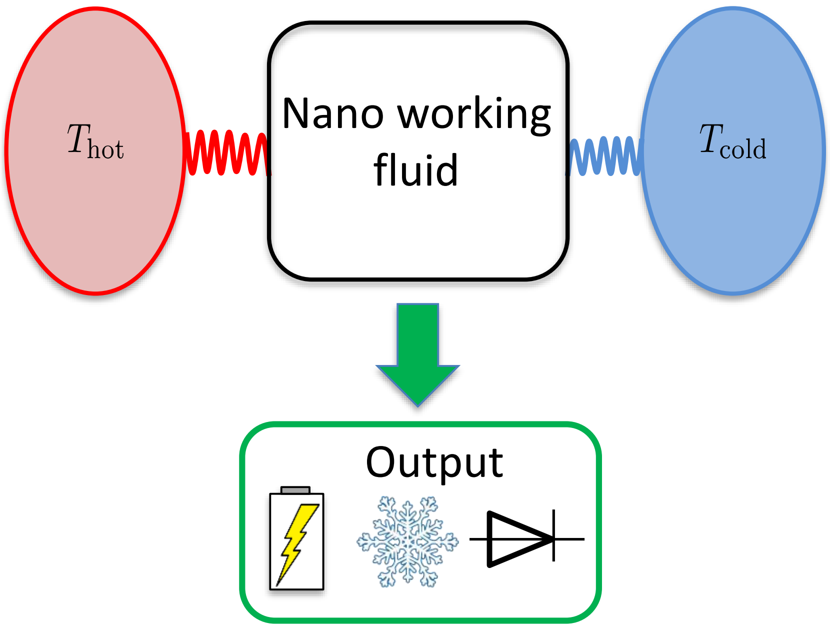


Thermal machine (XXIth)

SwissMAP general meeting

Saanenmöser, 08.09.2020

What can be quantum in thermal machines ?

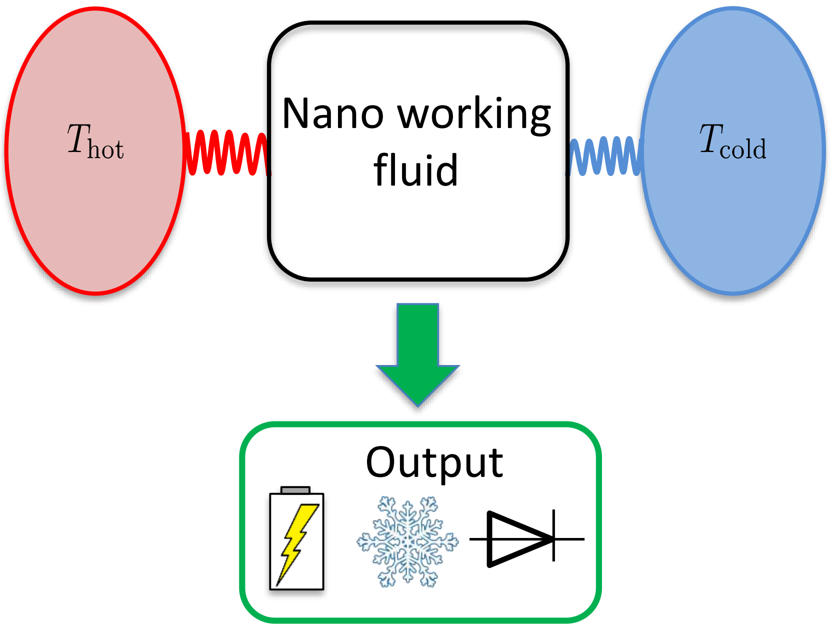


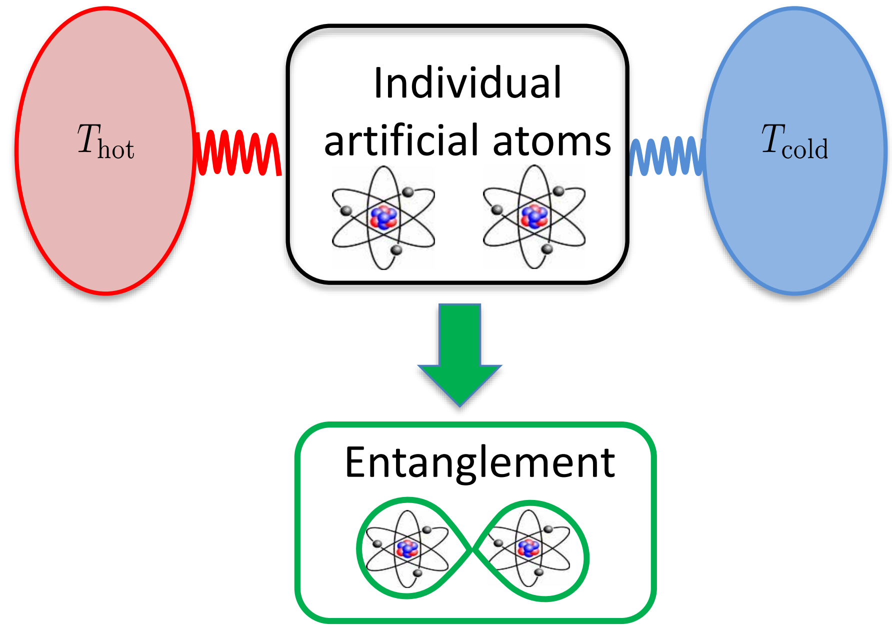
Few exemples

- Energy quantization (Exp: Linke)
- Many-body effects (Goold, Haack)
- Quantum phase coherence (Samuelson, Sothmann, Haack)

Is there a quantum advantage?

What can be quantum in thermal machines ?





Few exemples

- Energy quantization (Exp: Linke)
- Many-body effects (Goold, Haack)
- Quantum phase coherence (Samuelson, Sothmann, Haack)

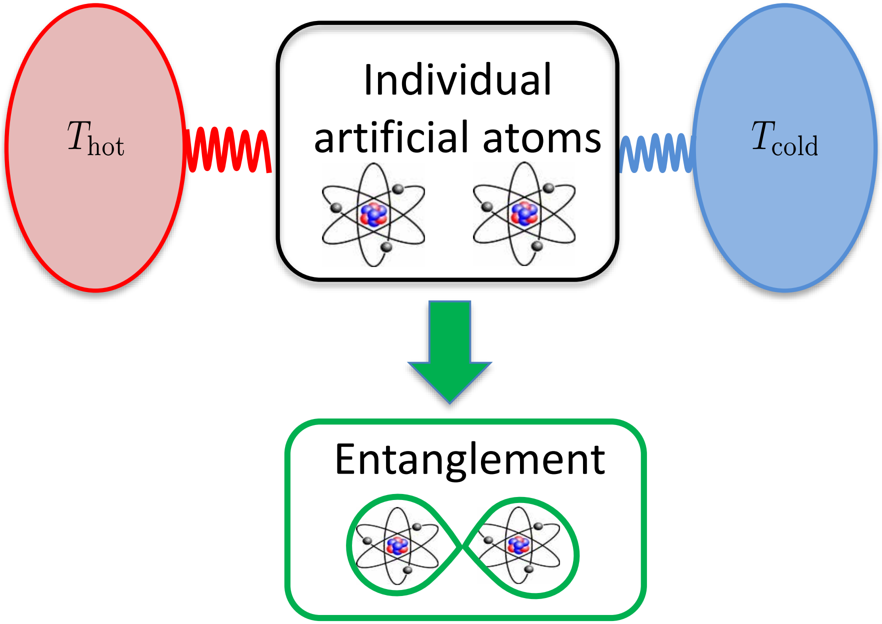
Is there a quantum advantage?

Some specificities

- Out of equilibrium quantum systems
- Exploiting incoherent thermal resources to create quantum correlations
- “Output” is quantum

Exploit dissipation for genuine Q. outputs

What can be quantum in thermal machines ?

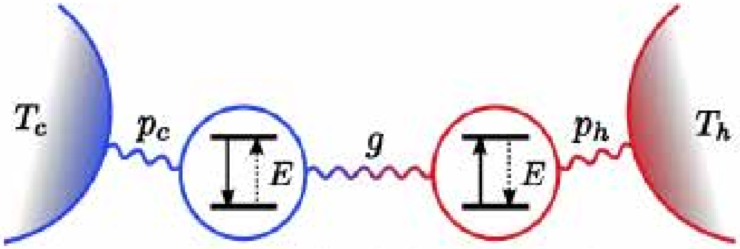


New motivations

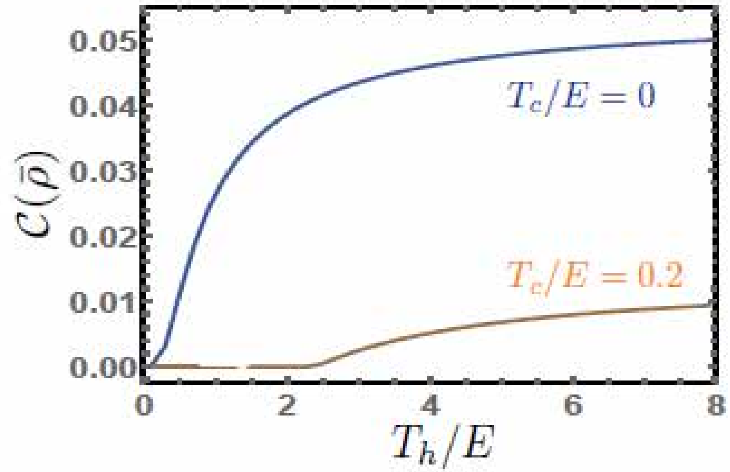
- Minimal models for out-of-equilibrium systems
- Dynamics of Open Q. systems
- Q. Thermodynamics <-> Q. information
- From system Q. properties to observables (quantum transport)

Out-of-equilibrium entanglement engines

- Most basic model



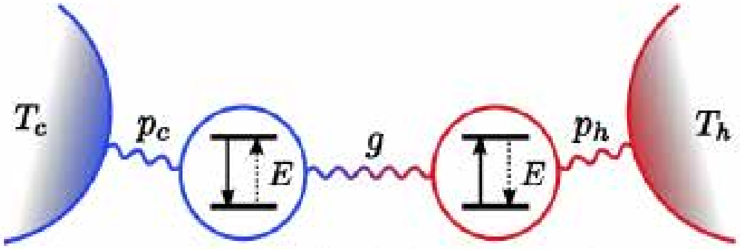
$$H_{int} = g\left(|01\rangle\langle 10| + h.c.\right)$$



Brask*, Haack*, Brunner, Huber, NJP 17 (2015)

Out-of-equilibrium entanglement engines

- Most basic model



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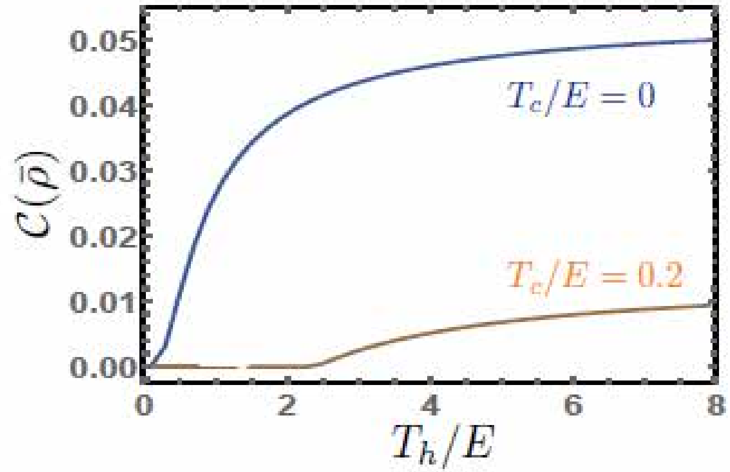


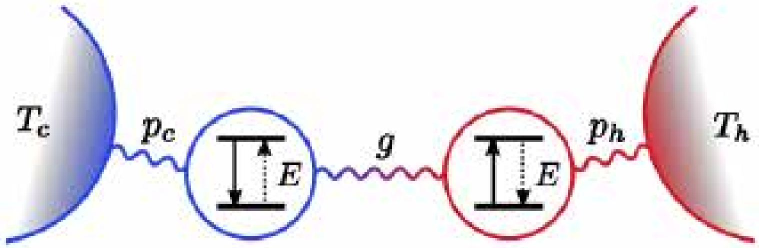
Brask*, Haack*, Brunner, Huber, NJP 17 (2015)

- Various systems:
  - Atom coupled to cavities driven by incoherent light
  - Qubits subject to noisy channel
  - Mechanical oscillators
- Specific interaction Hamiltonian (Ising-type, XX-type, ...)

Plenio, Huelga, PRL 88 (2002)
Eisler, Zimboras, PRA 71 (2005)
Hartmann et al., NJP 9 (2007)
Quiroga et al., PRA 75 (2007)
Linden et al., PRL 105 (2010)
Bellomo et a., NJP 15 (2013)
Brunner et al., PRE 89 (2014)
Brask et al., NJP 17 (2015)
Boyanovsky et al., PRA 96 (2017)

Generation of steady-state entanglement
Using only incoherent couplings to thermal baths

Out-of-equilibrium entanglement engines



$$H_s = E(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2)$$
$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

- Time-independent interaction Hamiltonian, time-independent bath couplings

  $\longrightarrow$ Thermodynamics: no work, only heat exchange

- Autonomous quantum thermal machine

- Ground state is a product state when $g < E$ (weak inter-qubit coupling)

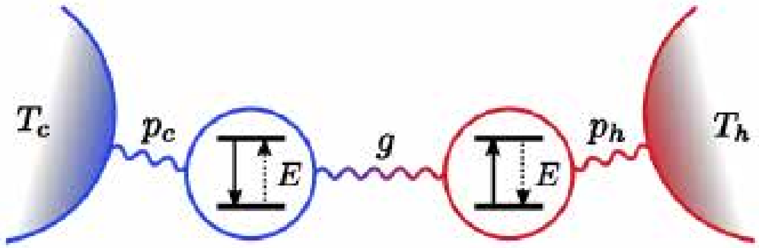
  In the energy eigenbasis $\{|00\rangle, |\Psi_-\rangle, |\Psi_+\rangle, |11\rangle\}$,

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E-g & 0 & 0 \\ 0 & 0 & E+g & 0 \\ 0 & 0 & 0 & 2E \end{pmatrix}$$

- Solve master equation to obtain the steady-state solution

Brask, Haack, Brunner, Huber, NJP 17 (2015)

Quantum dynamics



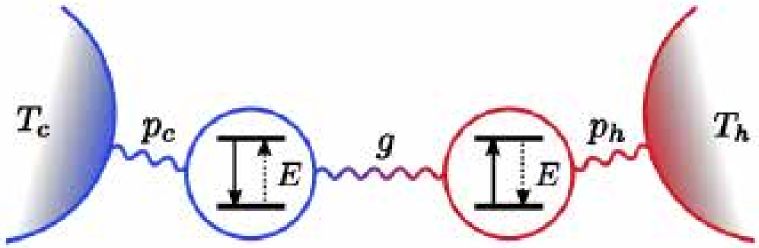
$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$
$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

- Probabilistic reset: $\rho(t+dt) = -i[H_s, \rho(t)]\,dt + \gamma dt\,\tau + (1-\gamma dt)\rho(t)$

  Thermal state $\quad \tau = r|0\rangle\langle 0| + (1-r)|1\rangle\langle 1|$

  Ground state population $\quad r = \dfrac{1}{1+e^{-E/(k_B T)}}$

Quantum dynamics



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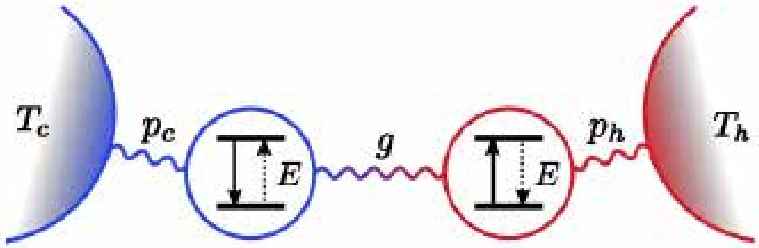
- Reset master equation (local): $\dot{\rho}(t) = -i[H_s, \rho(t)] + \gamma\,(\tau - \rho(t))$

Quantum dynamics



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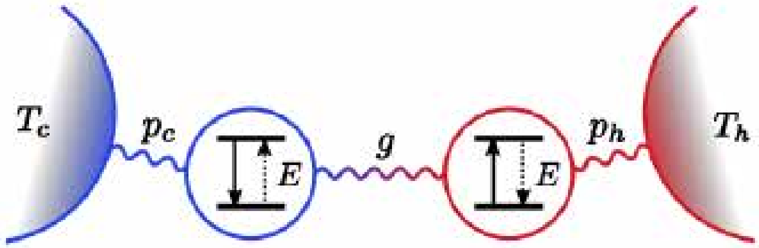
- For two qubits: $\dot\rho(t) = -i[H_s + H_{int}, \rho(t)] + \gamma_h\,(\tau_h \otimes \mathrm{Tr}_h\rho(t) - \rho(t)) + \gamma_c\,(\mathrm{Tr}_c\rho(t) \otimes \tau_c - \rho(t))$
- Analytic steady-state

$$\bar\rho = \begin{pmatrix} X & 0 & 0 & 0 \\ 0 & X & X & 0 \\ 0 & X & X & 0 \\ 0 & 0 & 0 & X \end{pmatrix} = \gamma\left[p_c p_h \tau_c \otimes \tau_h + \frac{2g^2}{(p_c+p_h)^2}(p_c\tau_c + p_h\tau_h)^{\otimes 2} + \frac{g p_c p_h (r_c - r_h)}{p_c + p_h}\mathcal{Y}\right]$$

Quantum dynamics



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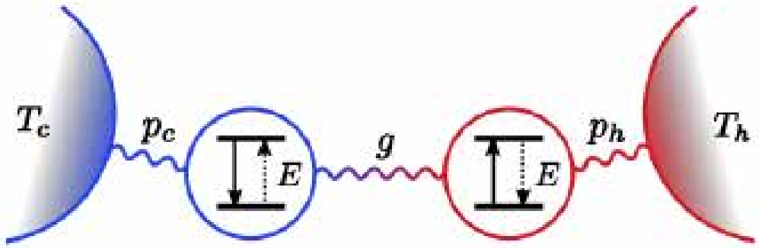
Perturbative analysis of quantum reset models in a tri-partite configuration
G. Haack & A. Joye, arXiv:2009.03054

Entanglement engine



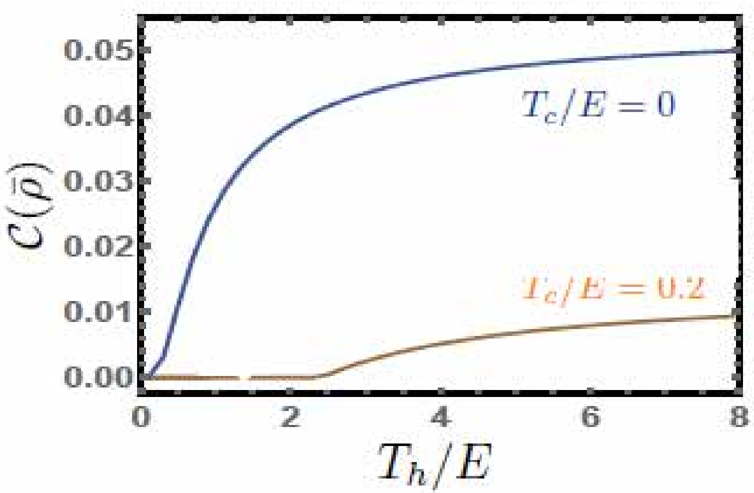
$$H_s = E(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2)$$
$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

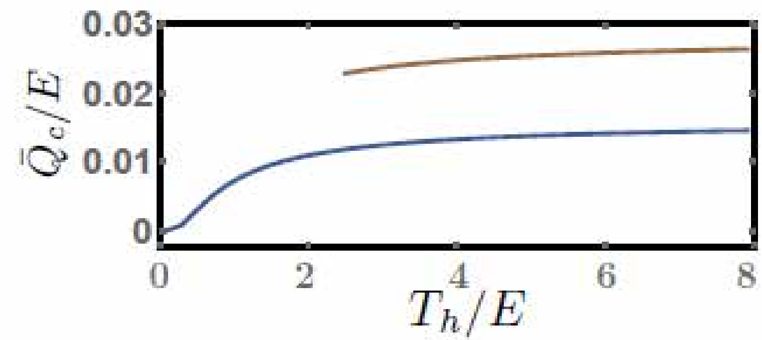
- Concurrence (measure of entanglement) : Given by the eigenvalues of $R = \sqrt{\sqrt{\bar{\rho}}\tilde{\bar{\rho}}\sqrt{\bar{\rho}}}$

Wooters, PRL (2001)

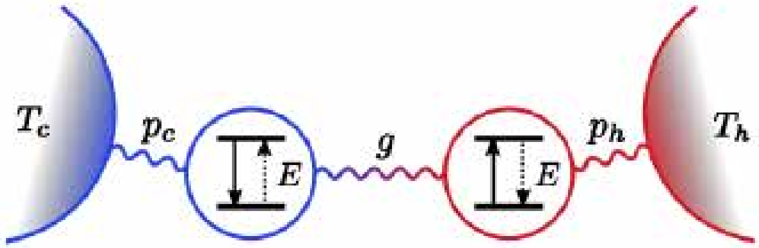
- Heat flow: $\bar{Q}_c = p_c E\langle 1|\bar{\rho}_c - \tau_c|1\rangle$

(Parameters optimization for each temp. bias)





Entanglement engine



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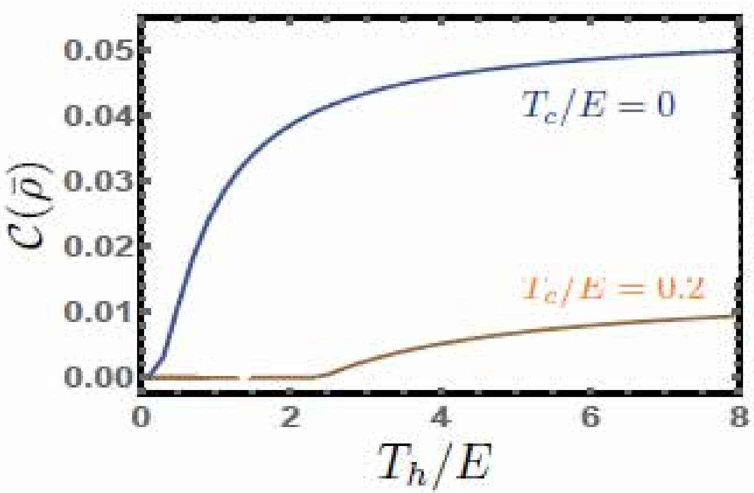
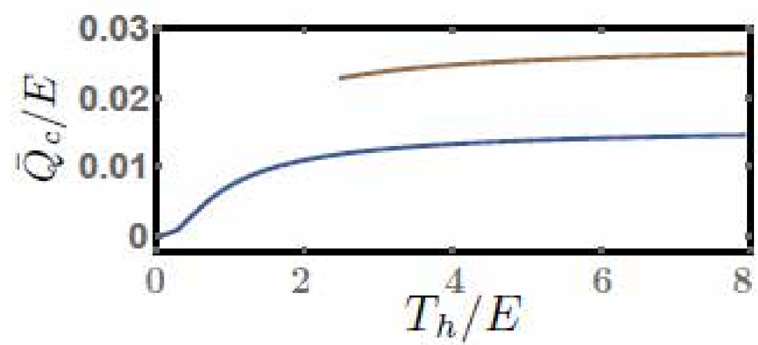
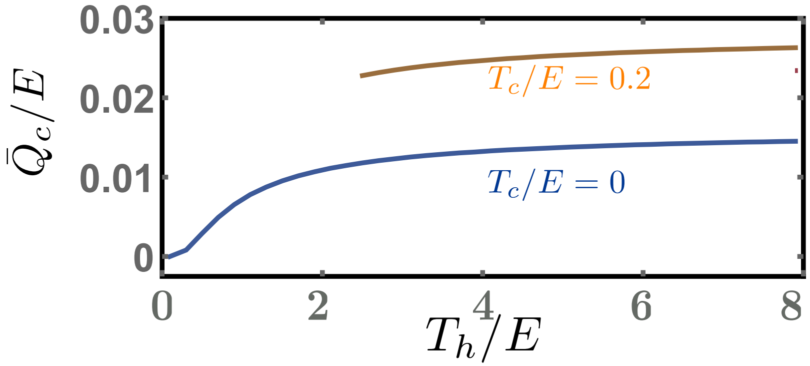




Quantum information measure ⟷ Quantum transport

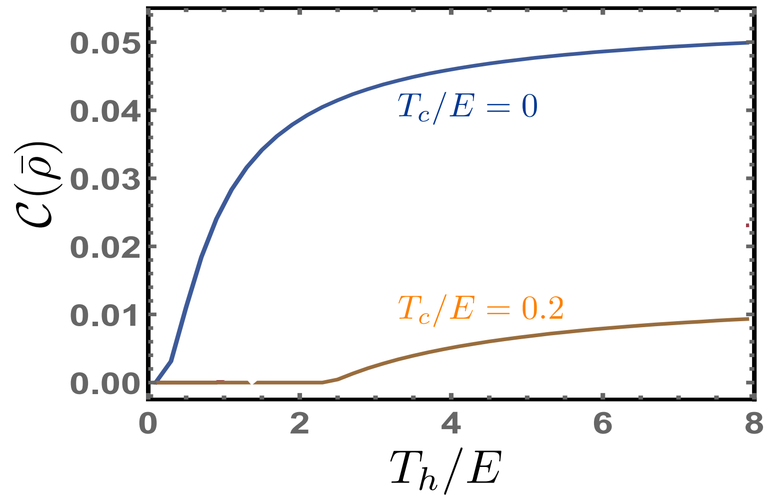
New questions

Heat flow to the cold bath (-)



Brask, Haack, Brunner, Huber, NJP **17**, 113029 (2015)

Concurrence



-> Are thermal resources a fundamental limitation to generate useful quantum correlations?

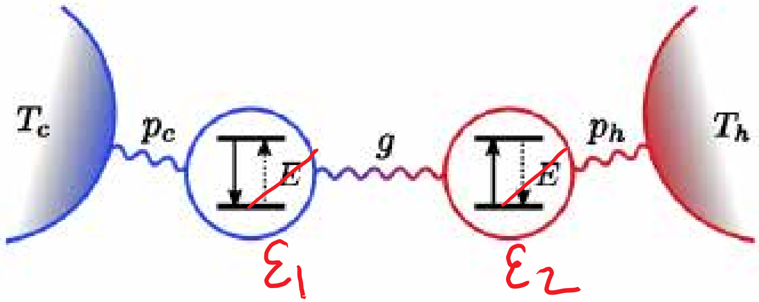
Heralded entanglement engines : bipartite Tavakoli et al., Quantum 2 (2018)
multipartite Tavakoli et al., PRA 101 (2020)

-> Is there a relation between heat current and entanglement measures ?

Thermo <-> quantum info
Q. tomography <-> observables

Khandelwal, Palazzo, Brunner, Haack, New J. Phys. 22 (2020)

Model



$$H_s = \cancel{E}\,\varepsilon_1(|1\rangle\langle 1|_1 + \varepsilon_2|1\rangle\langle 1|_2)$$
$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$

- Steady-state regime

ALSO STRONG global

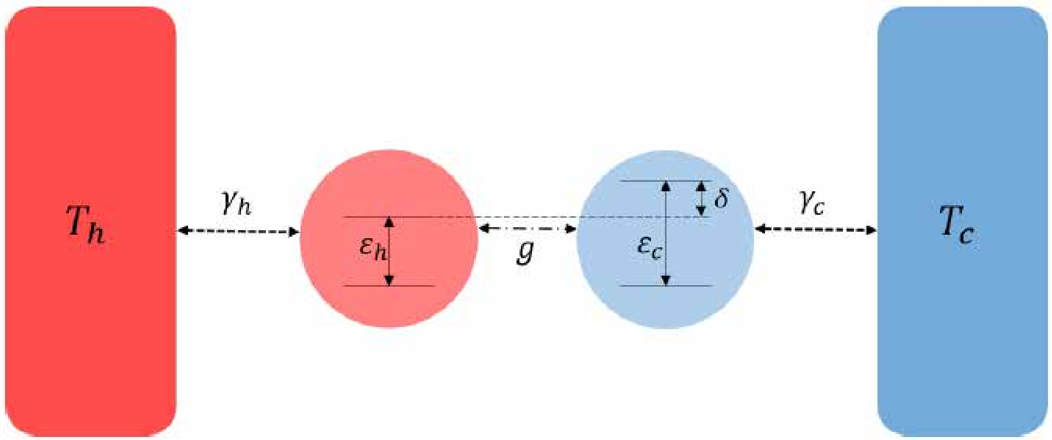
- Weak inter-qubit interaction -> ground ~~state is~~ separable + “~~local~~al” master equation
- Reset master equation (probabilistic reset of each qubit to the thermal state corresponding to its bath, incoherent coupling ~ Lindblad dissipators)

↳ LINDBLAD ME

- Quantities : ~~heat flow~~ to the cold bath and ~~concurrence~~ as a measure of entanglement

HEAT CURRENT NEGATIVITY (BUT EQUIVALENT FOR THIS MODEL)

Model in the weak inter-qubit coupling regime



$$H = H_{\mathrm{S}} + H_{\mathrm{int}} + H_{\mathrm{B}} + H_{\mathrm{SB}}$$

$$H_{\mathrm{S}} = \sum_{j\in\{h,c\}} \varepsilon_j \sigma_+^{(j)}\sigma_-^{(j)} \qquad H_{\mathrm{int}} = g\left(\sigma_+^{(h)}\sigma_-^{(c)} + \sigma_-^{(h)}\sigma_+^{(c)}\right)$$

$$H_{\mathrm{B}} = \sum_{j\in\{h,c\}} \omega_j c_j^\dagger c_j \qquad H_{\mathrm{SB}} = \sum_{j\in\{h,c\}} \left(\alpha_j \sigma_-^{(j)} c_j^\dagger + \alpha_j^* \sigma_+^{(j)} c_j\right)$$

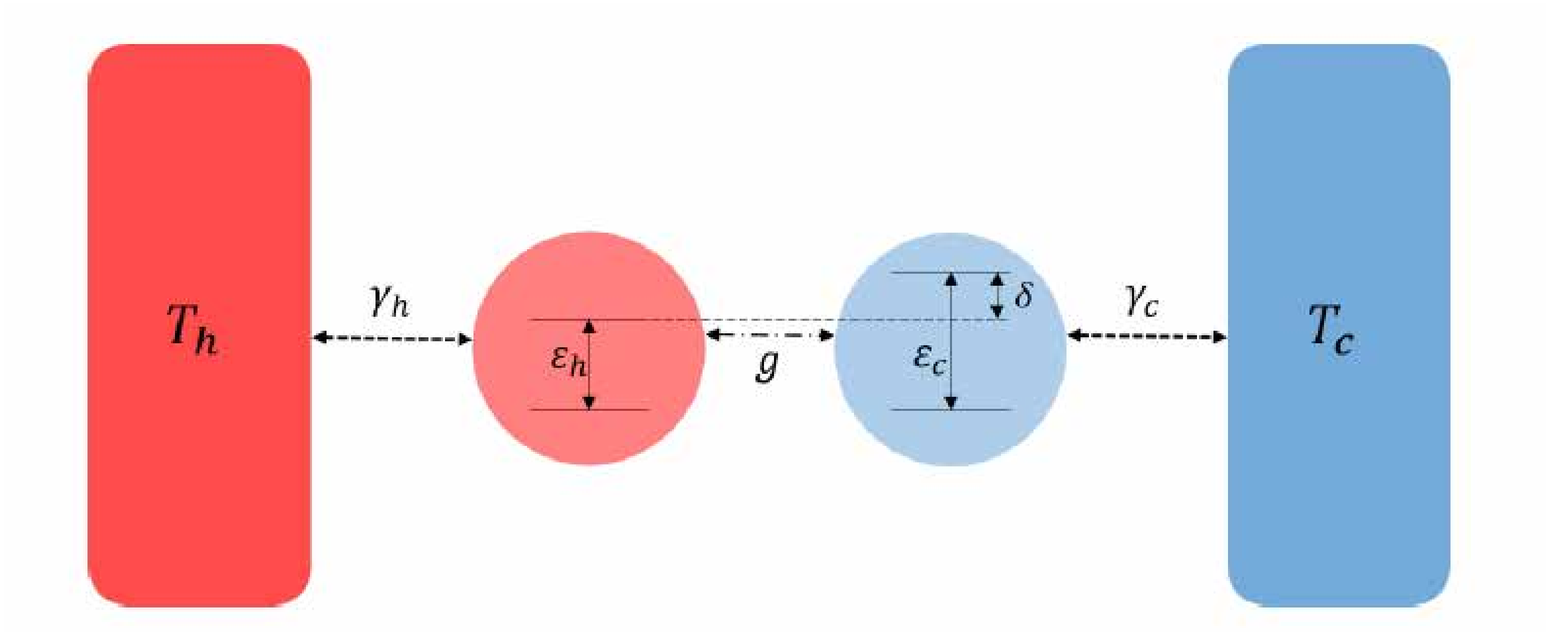
(Local) master equation

operators acting on the individual $\mathcal{H}$ of the qubits

$$\begin{aligned}\dot{\rho}(t) &= \mathcal{L}\rho(t) \\ &= -i[H_{\mathrm{S}} + H_{\mathrm{int}}, \rho(t)] + \sum_{j\in\{h,c\}} \gamma_j^+ \mathcal{D}\left[\sigma_+^{(j)}\right]\rho(t) + \gamma_j^- \mathcal{D}\left[\sigma_-^{(j)}\right]\rho(t)\end{aligned}$$

Khandelwal, Palazzo, Brunner, Haack, New J. Phys. 22 (2020)

Results



Steady-state solution

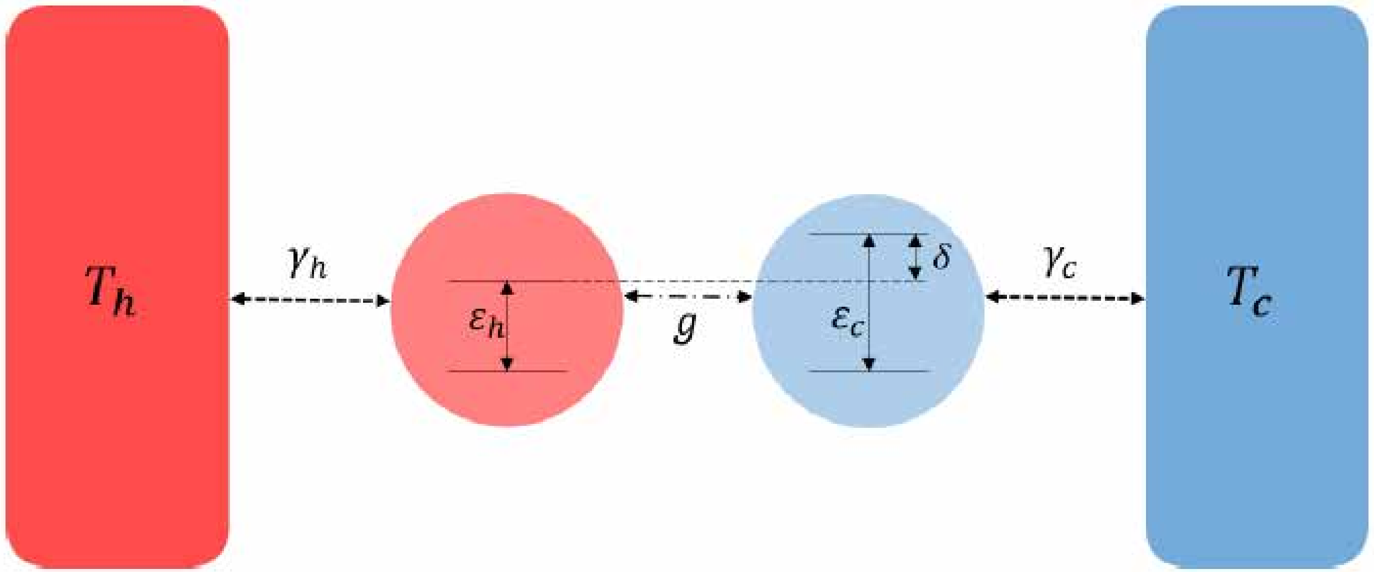
$$\rho_{ss} = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & c & 0 \\ 0 & c^* & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix}$$

$$c = \frac{2g\,(i\Gamma - 2\delta)}{\chi}\,\gamma_c\gamma_h\,\Big(\underbrace{n_h(\varepsilon_h, T_h) - n_c(\varepsilon_c, T_c)}_{\text{difference of BE bath distr.}}\Big)$$

$$J_{ss} = \frac{8g^2\,(\varepsilon_h\Gamma_c + \varepsilon_c\Gamma_h)}{\chi}\,\gamma_c\gamma_h\,\big(\overbrace{\;\cdots\;}\big)$$

At thermal equilibrium -> no heat current, no coherence in the steady-state regime

Critical heat current for successful operation of the machine



Measure of entanglement : negativity

$$N(\rho) := \sum_{\lambda_i<0} |\lambda_i| \quad \in [0;0.5]$$

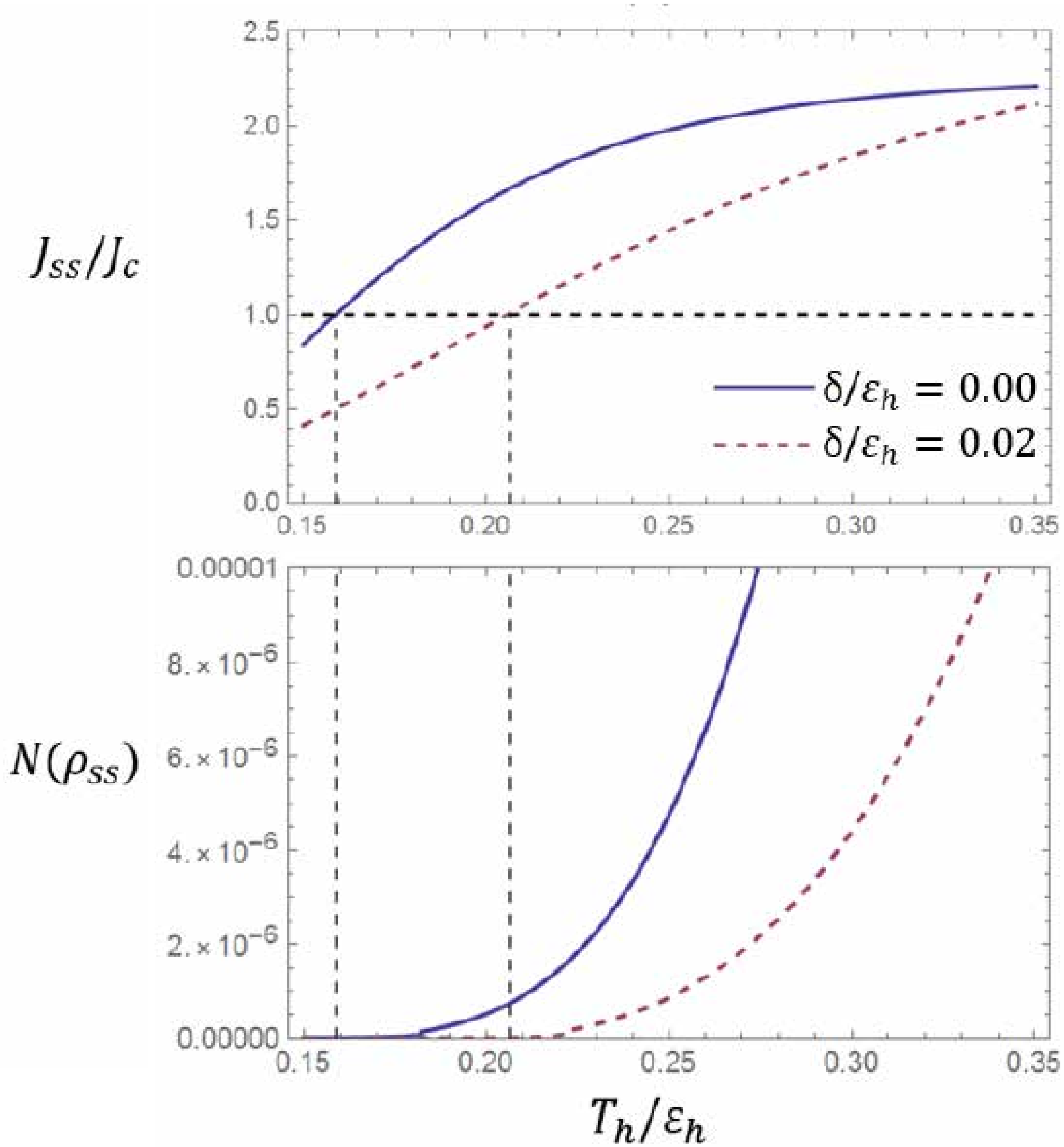
Eigenvalues of the partial transpose of the DM wrt one of the qubits

$$\rho_{ss} = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & c & 0 \\ 0 & c^* & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{pmatrix} \Rightarrow N(\rho) \neq 0 \Leftrightarrow |c|^2 > r_1 r_4$$

J_{ss} is a fonction of c $\rightarrow$ J_{crit}

Exact relation between quantum correlations' measure and transport observable

Critical heat current for successful operation of the machine



-> heat-based entanglement witness (alternative to quantum tomography?)

Perspectives - motivations

- Minimal models for quantum (mesoscopic) thermal machines
  (thermal diodes, entanglement engines, thermoelectric engines)

- Development of theoretical tools for assessing their dynamics
  (quantum trajectories, perturbative analysis, strong coupling regime)

- Q. Thermodynamics <-> Q. information <-> Q. transport

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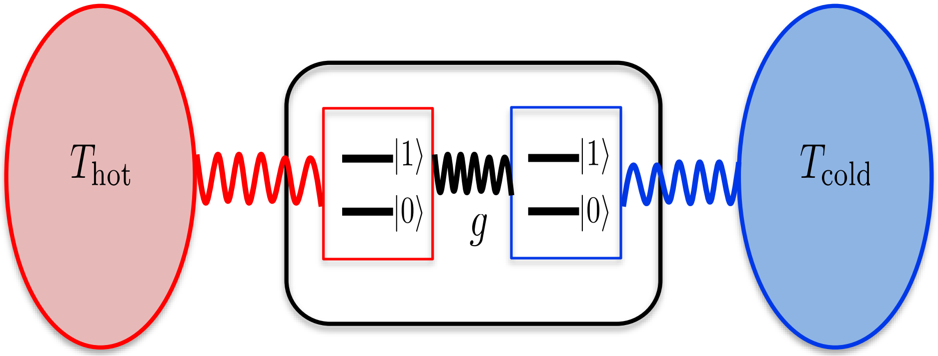
S. Khandelwal, N. Palazzo, N. Brunner, G. Haack, New J. Phys. 22 (2020)
G. Haack & A. Joye, arXiv:2009.03054

Strong inter-qubit interaction regime



$$H = H_S + H_{int} = \begin{pmatrix} 2\varepsilon & & & \\ & \varepsilon & g & \\ & g & \varepsilon & \\ & & & 0 \end{pmatrix}$$

- Thermal state of two qubits at temperature T

$$\rho_{\rm th} = \frac{e^{-(H_{\rm S}+H_{\rm int})/T}}{\mathrm{Tr}\left(e^{-(H_{\rm S}+H_{\rm int})/T}\right)}$$

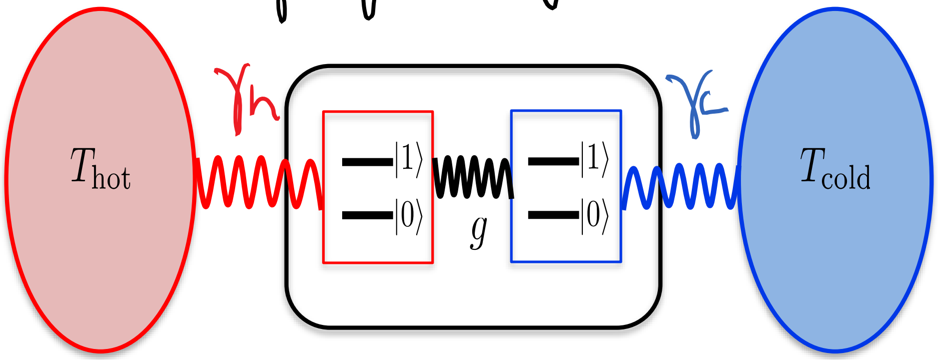
$$= \frac{1}{2\left(\cosh\left(\frac{\varepsilon}{T}\right)+\cosh\left(\frac{g}{T}\right)\right)} \begin{pmatrix} e^{-\varepsilon/T} & 0 & 0 & 0 \\ 0 & \cosh\left(\frac{g}{T}\right) & -\sinh\left(\frac{g}{T}\right) & 0 \\ 0 & -\sinh\left(\frac{g}{T}\right) & \cosh\left(\frac{g}{T}\right) & 0 \\ 0 & 0 & 0 & e^{\varepsilon/T} \end{pmatrix}$$

- Its negativity

$$N\left(\rho_{\rm th}\right) = \max\left\{0, \frac{\sqrt{\cosh^2\left(\frac{\varepsilon}{T}\right)+\sinh^2\left(\frac{g}{T}\right)-1}-\cosh\left(\frac{\varepsilon}{T}\right)}{2\left(\cosh\left(\frac{\varepsilon}{T}\right)+\cosh\left(\frac{g}{T}\right)\right)}\right\} > 0 \Leftrightarrow \sinh^2(g/T) > 1$$

Strong inter-qubit interaction regime

$\gamma_h, \gamma_c \ll g < \varepsilon$



$$H = H_s + H_{int}$$

$$= \begin{pmatrix} 2\varepsilon & & & \\ & \varepsilon & g & \\ & g & \varepsilon & \\ & & & 0 \end{pmatrix}$$

- Dynamics: global master equation

-> jump operators act onto the eigenstates of the two-qubit system

$$\dot{\rho}(t) = -i\left[H_{\text{S}} + H_{\text{int}}, \rho(t)\right] + \sum_{\alpha \in \{-,+\}} \sum_{j \in \{h,c\}} \gamma_j^{-}(\varepsilon_\alpha)\, \mathcal{D}\left[\hat{L}_j(\varepsilon_\alpha)\right]\rho(t) + \gamma_j^{+}(\varepsilon_\alpha)\, \mathcal{D}\left[\hat{L}_j^{\dagger}(\varepsilon_\alpha)\right]\rho(t)$$

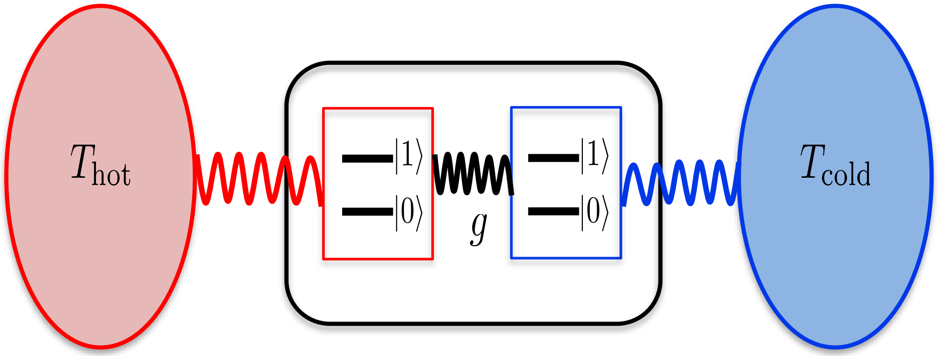
with:

$$\hat{L}_j(\varepsilon_-) = |0\rangle\langle 0|\sigma_-^{(j)}|\varepsilon_-\rangle\langle\varepsilon_-| + |\varepsilon_+\rangle\langle\varepsilon_+|\sigma_-^{(j)}|2\rangle\langle 2|$$
$$\hat{L}_j(\varepsilon_+) = |0\rangle\langle 0|\sigma_-^{(j)}|\varepsilon_+\rangle\langle\varepsilon_+| + |\varepsilon_-\rangle\langle\varepsilon_-|\sigma_-^{(j)}|2\rangle\langle 2|.$$

λ_i	$\lvert v_i\rangle$
0	$\lvert 0\rangle$
$\varepsilon_- = \varepsilon - g$	$\lvert \varepsilon_-\rangle$
$\varepsilon_+ = \varepsilon + g$	$\lvert \varepsilon_+\rangle$
2ε	$\lvert 2\rangle$

Punkayastha et al., PRA 93 (2016)
Hofer et al., NJP 19 (2017)
Gonzalez et al., Open S. Inf. D. 24 (2017)
Mitchison, Plenio, NJP 20 (2018)
De Chiara et al., NJP 20 (2018)
Cattaneo et al., NJP 21 (2019)

Strong inter-qubit interaction regime



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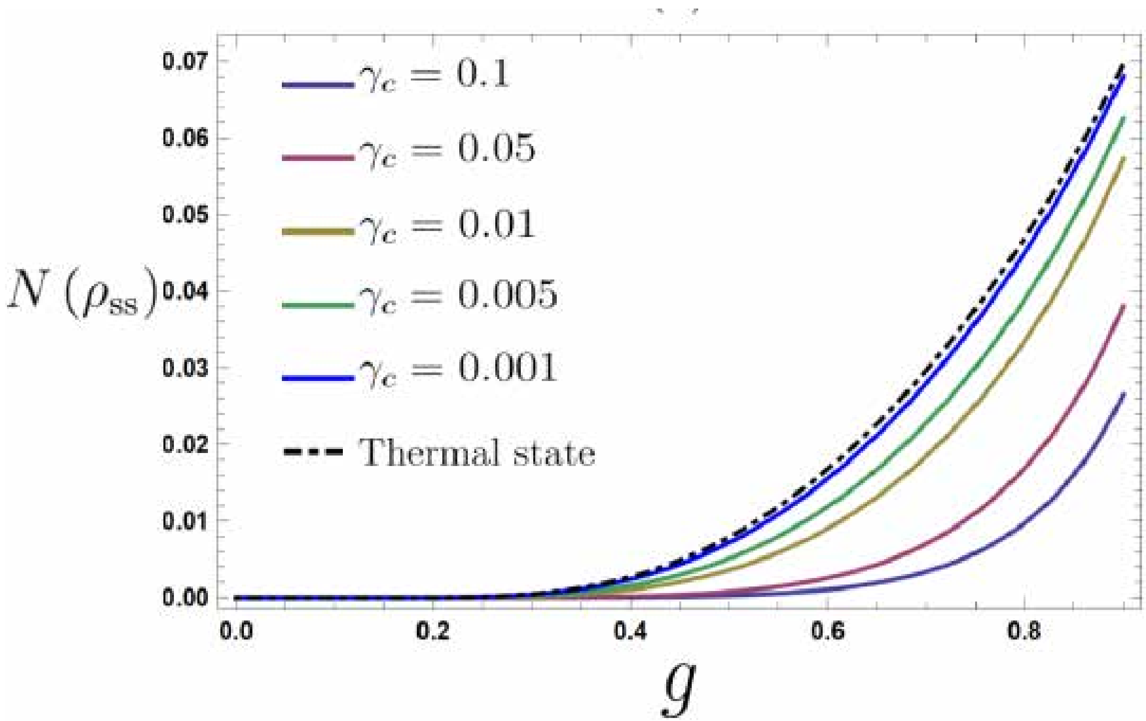
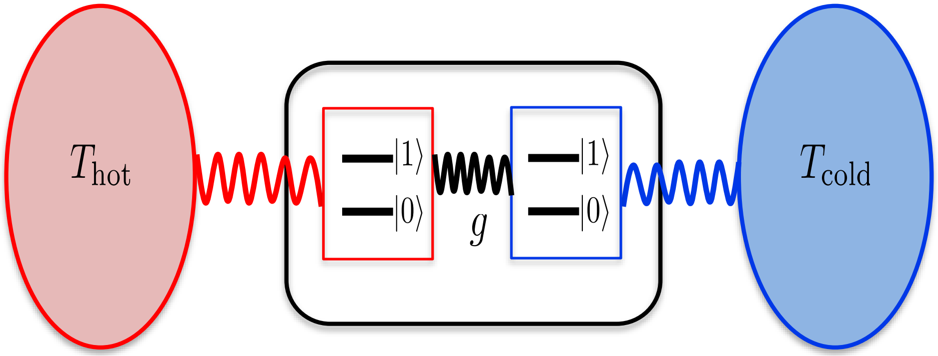
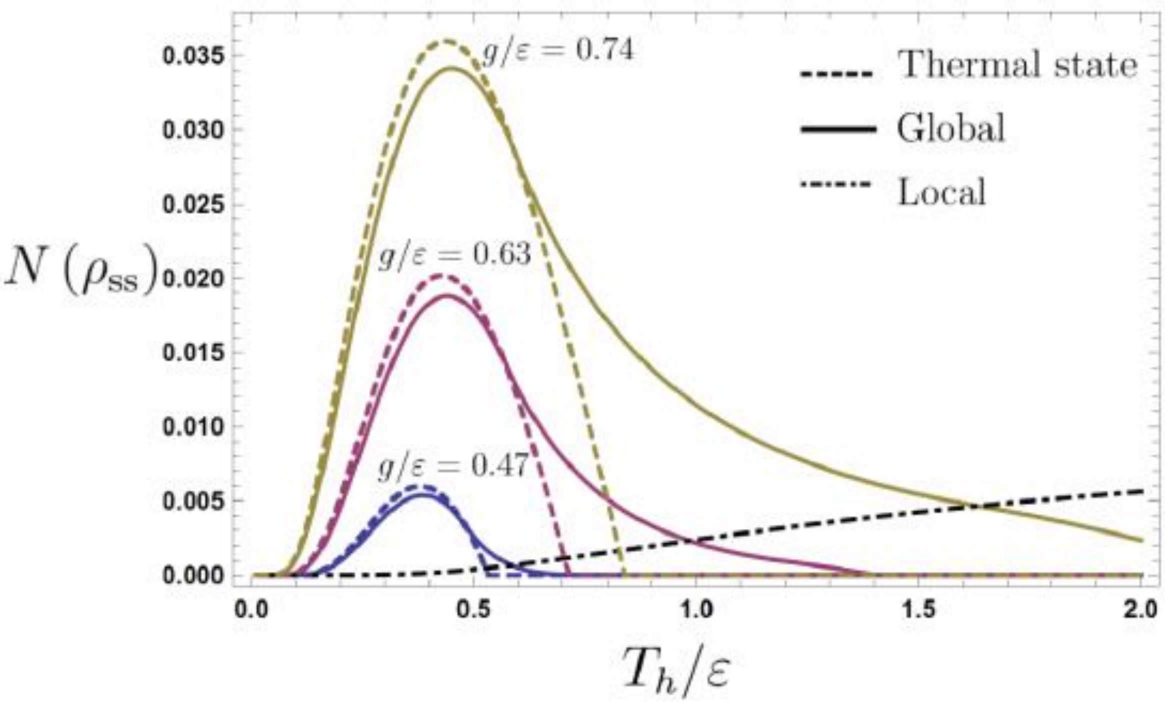


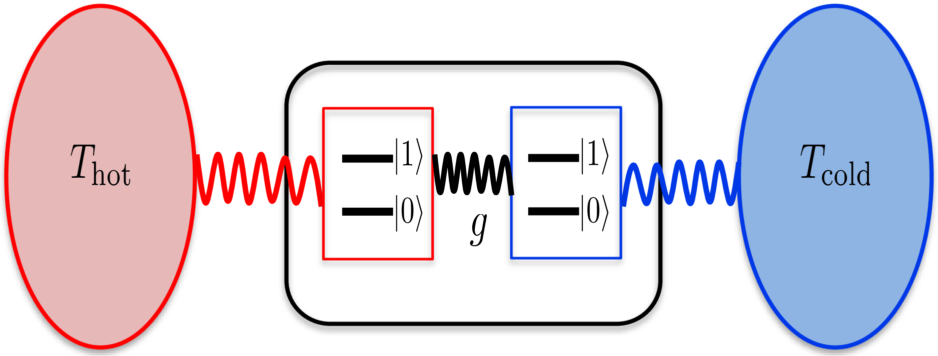
Strong inter-qubit interaction regime



$$H = H_s + H_{int}$$
$$= \begin{pmatrix} 2\varepsilon & & & \\ & \varepsilon & g & \\ & g & \varepsilon & \\ & & & 0 \end{pmatrix}$$



Strong inter-qubit interaction regime



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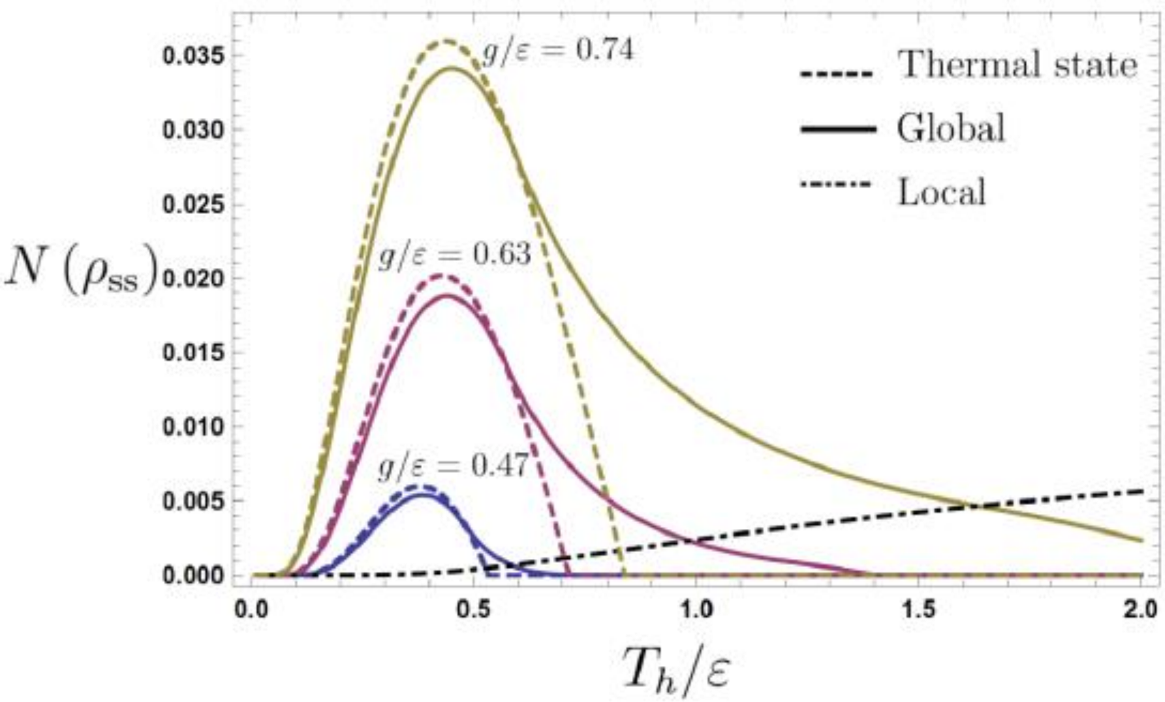

Global

- Critical heat current valid
- Sensitivity to T_h
- Bounded by thermal ent.
- No clear advantage wrt local