Ergodicity & Thermalisation in Low Dimensional theories

by

Pranjal Nayak

[based on 1903.00478 (JHEP 10 019), 1907.10061 (JHEP 03 168)
with Julian Sonner and Manuel Vielma
and ongoing work]
with JS, MV and Alex Altland

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

UNIVERSITÉ DE GENÈVE

SwissMap General Meeting

September 8, 2020

What is our work about?

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Quantum Mechanics is unitary!

$$|\psi(t)\rangle = \mathcal{U}(t, t_0)|\psi(t_0)\rangle$$

How come we observe
thermal physics?

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How come we observe
black hole formation?

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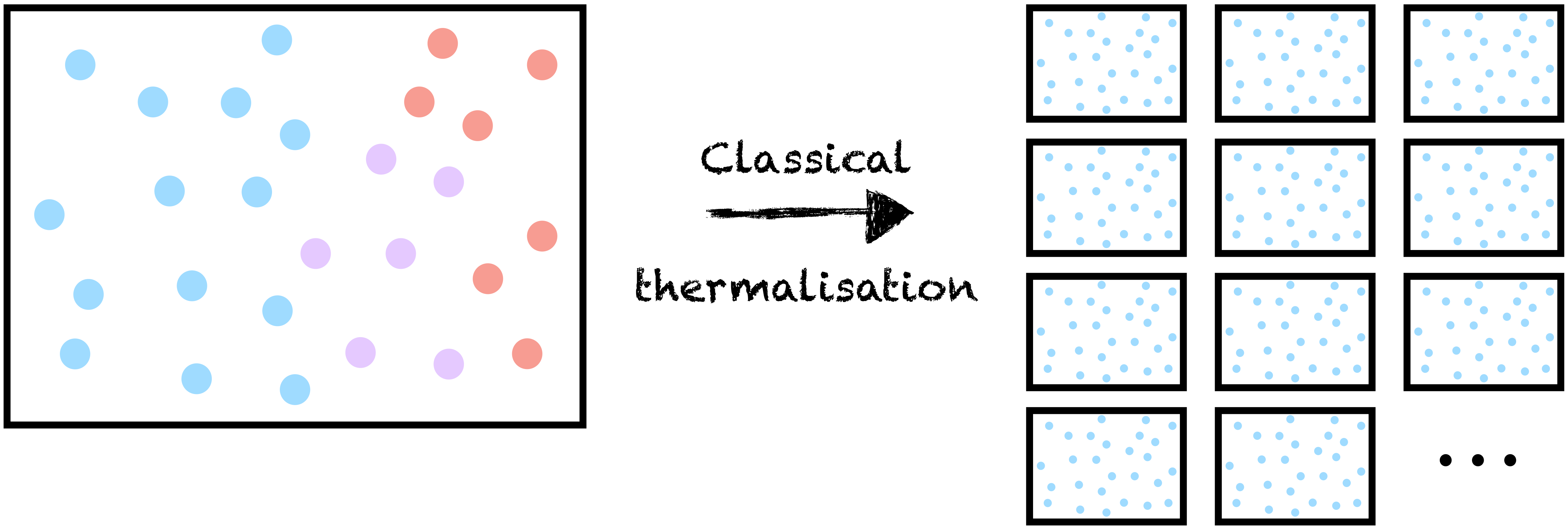

<u>Ergodic Hypothesis:</u> Over long periods of time, the time spend by a system in some region of phase space of micro states with same energy is proportional to the volume of the region.

A classical many-body system efficiently explores the *available* phase space

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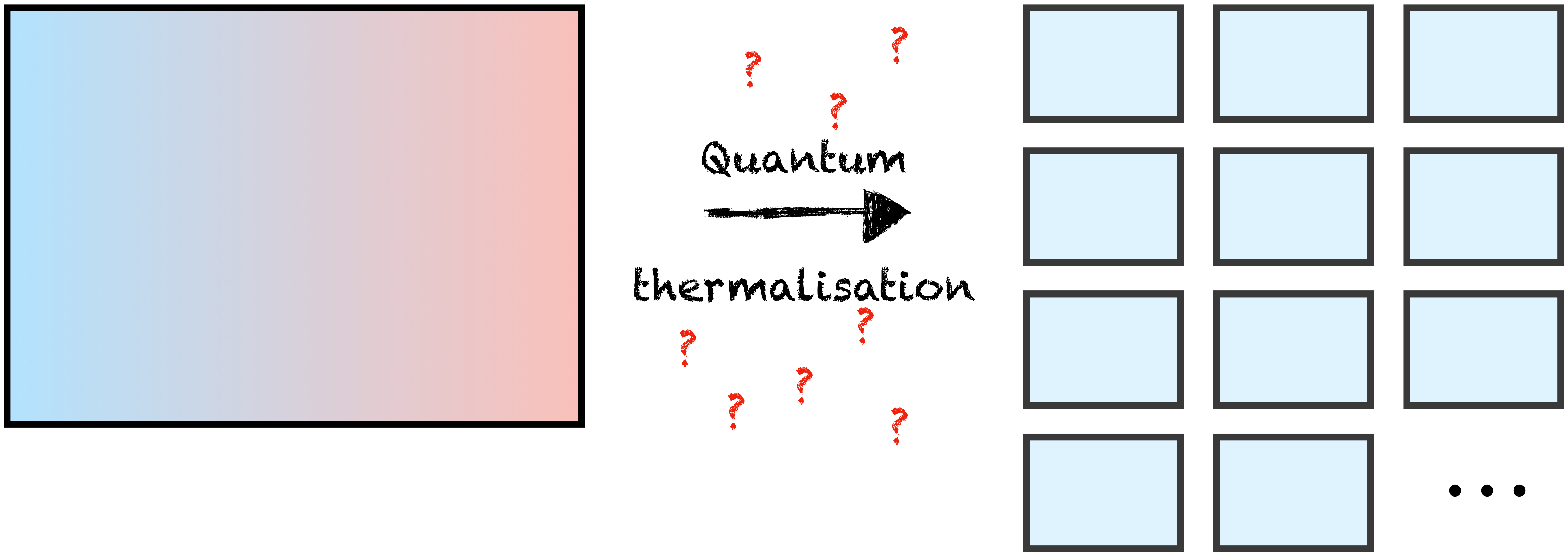

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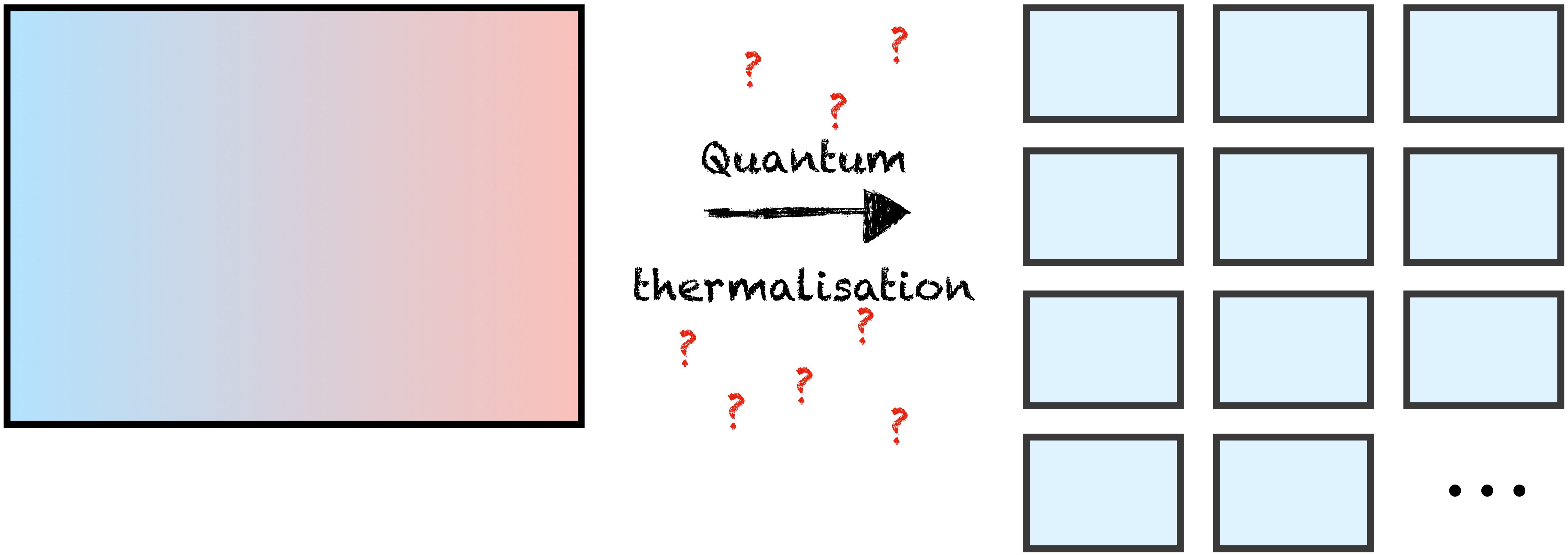

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$$\rho(t) = \sum_i p_i \, |\psi_i(t)\rangle\langle\psi_i(t)|$$

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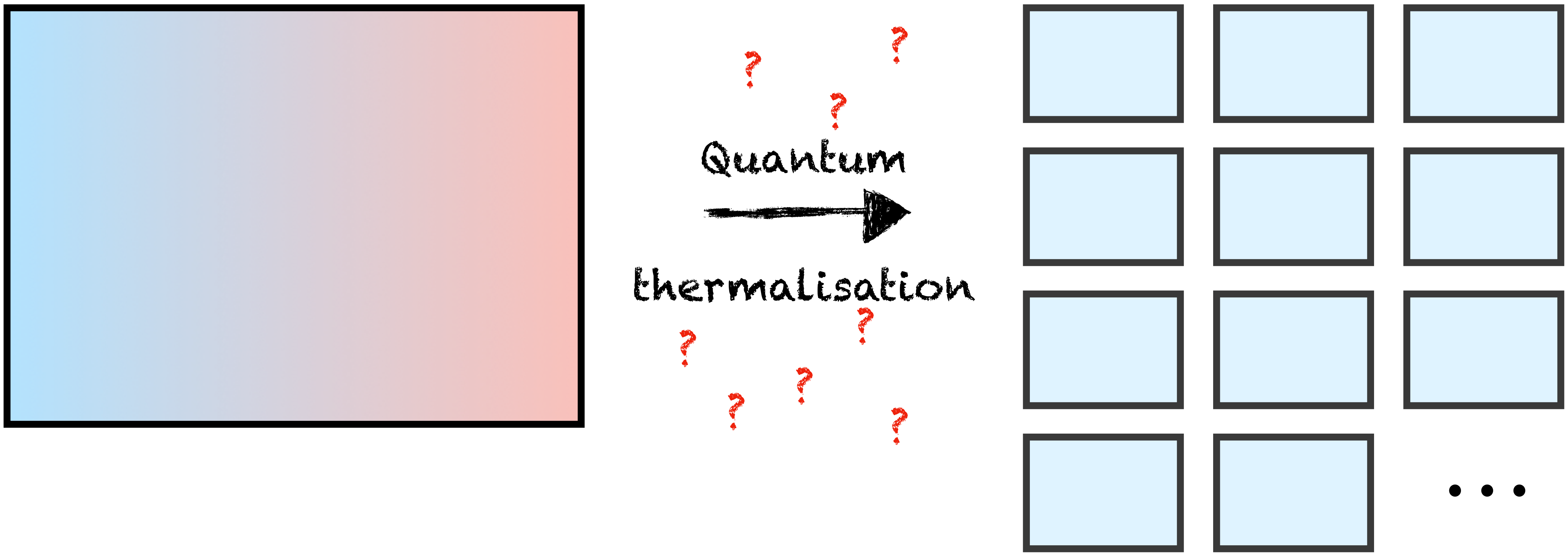

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Fundamentally different

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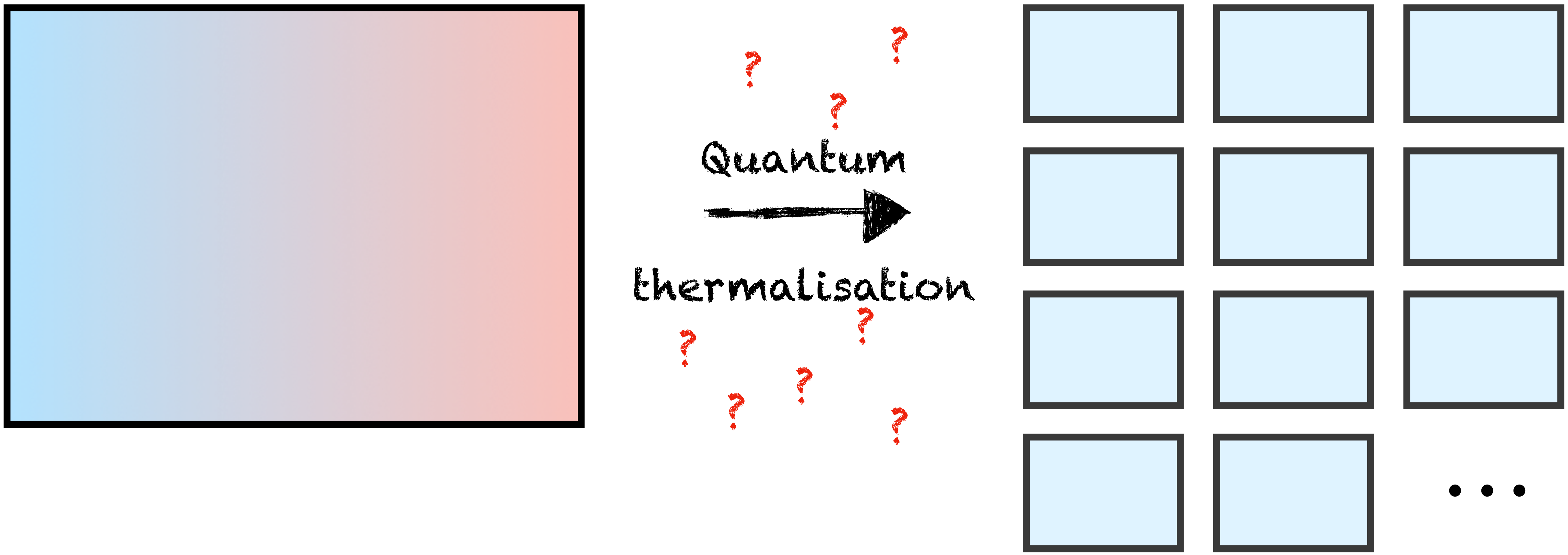

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Fundamentally different

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How do we understand thermalisation in Quantum Mechanics???

Quantum Thermalization

Quantum Thermalization

Quantum thermalization: Eigenstate Thermalisation Hypothesis

$$\langle m|\mathcal{O}|n\rangle = \overline{\mathcal{O}}_{\mathrm{mc}}(\overline{E})\delta_{mn} + e^{-S(\overline{E})/2} f(\overline{E},\omega) R_{mn}$$

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micro-canonical ensemble

avg. energy of ensemble

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[Srednicki '94]

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A generic excited state will then thermalise by **dephasing**

$$\langle\psi|\mathcal{O}|\psi\rangle = \sum_{i,j} c_i^* c_j e^{it(E_i - E_j)} \mathcal{O}_{ij} \longrightarrow \overline{O}\left(\bar{E}\right) + e^{-S}$$

expectation value of non-extensive operator

dephasing: spectral chaos

on average thermal up to exponential in S

What does it have to do with Black Holes?

semi-classical General Relativity

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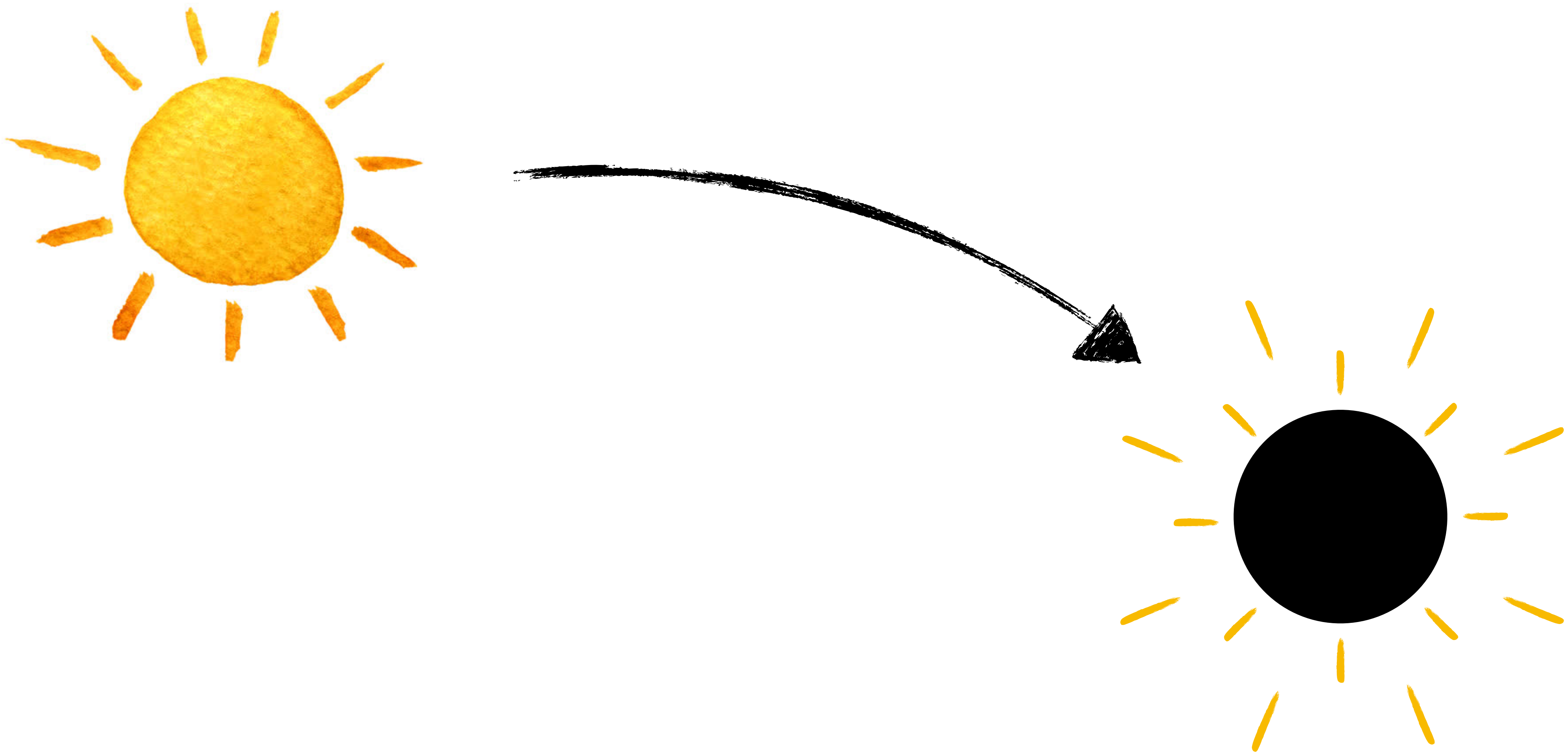

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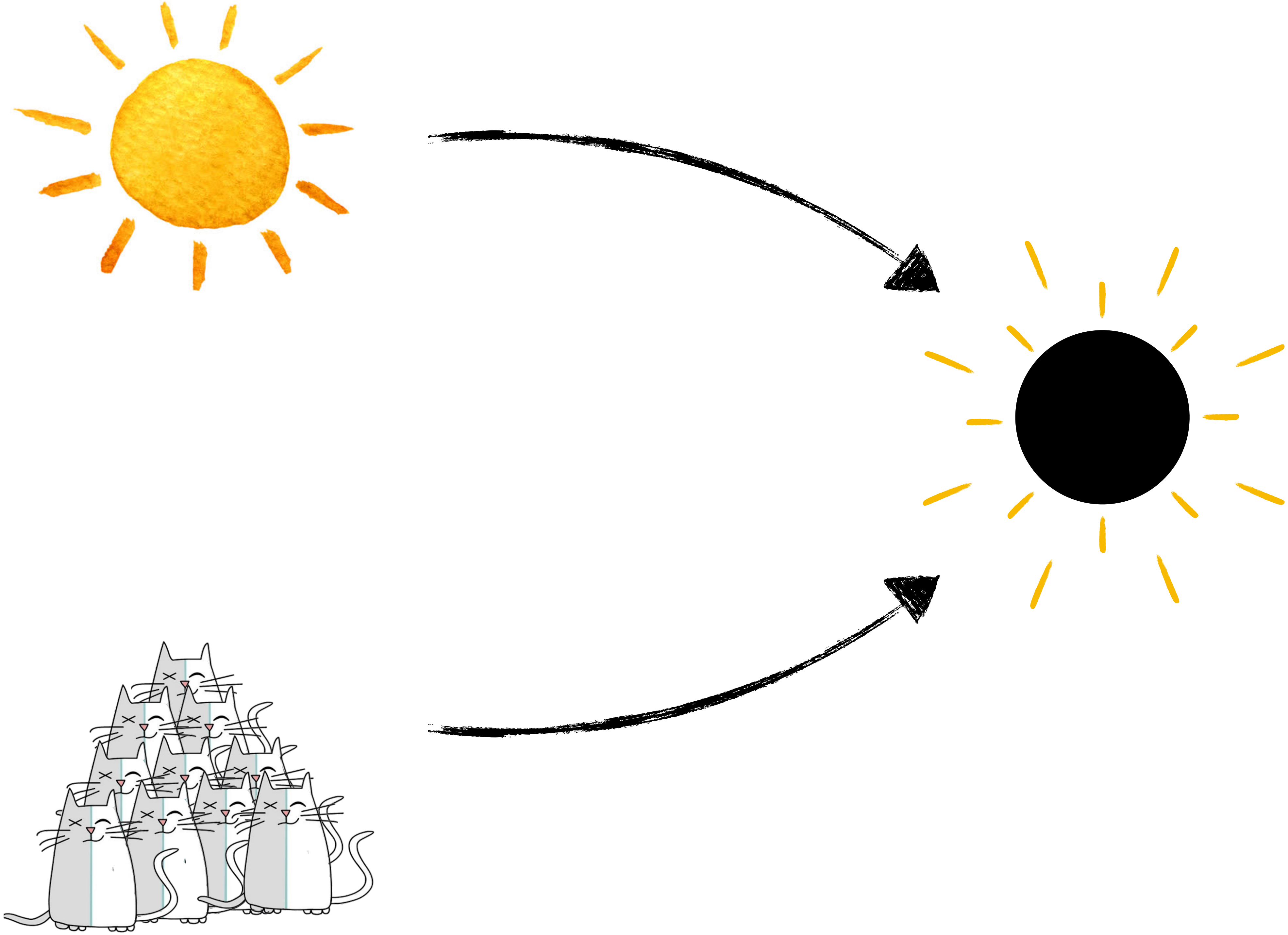

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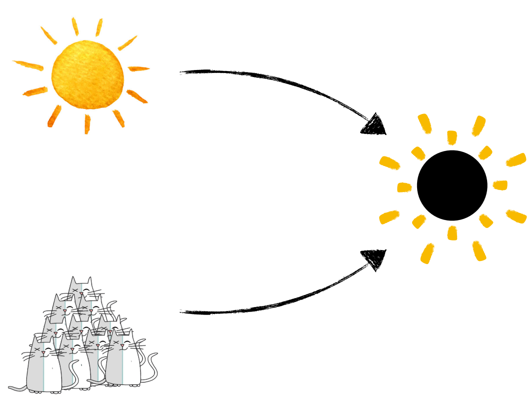
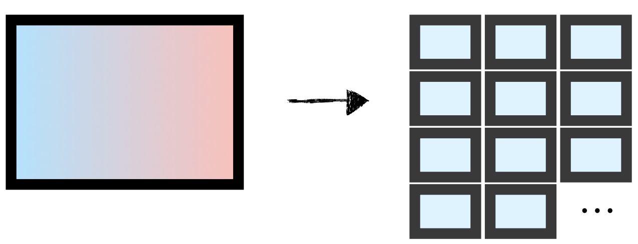
d+1 dimensional theory of gravity ⟷ d dimensional conformal field theory

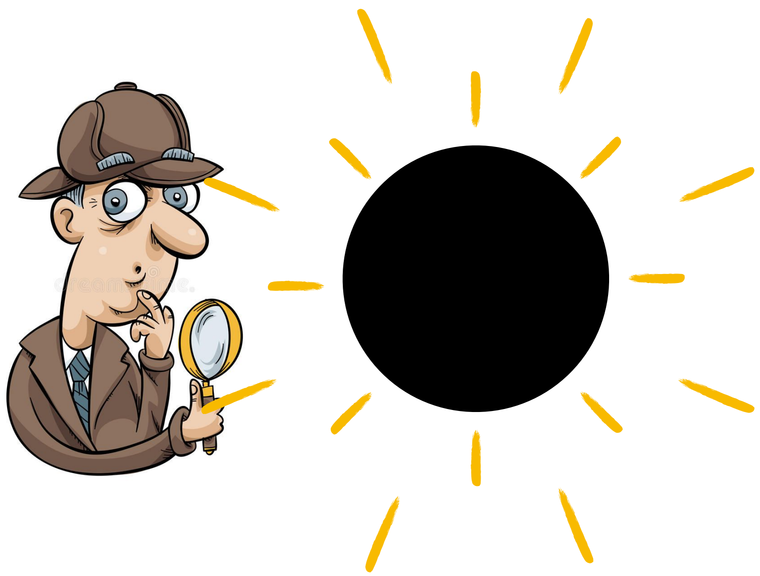




Black hole formation ⟷ Thermalisation

What does it have to do with Black Holes?

- Evidence from earlier works suggests that one needs to wait for extremely long times before we receive corrections from some sub-leading saddle point solutions of quantum gravity which *prevents* information loss.



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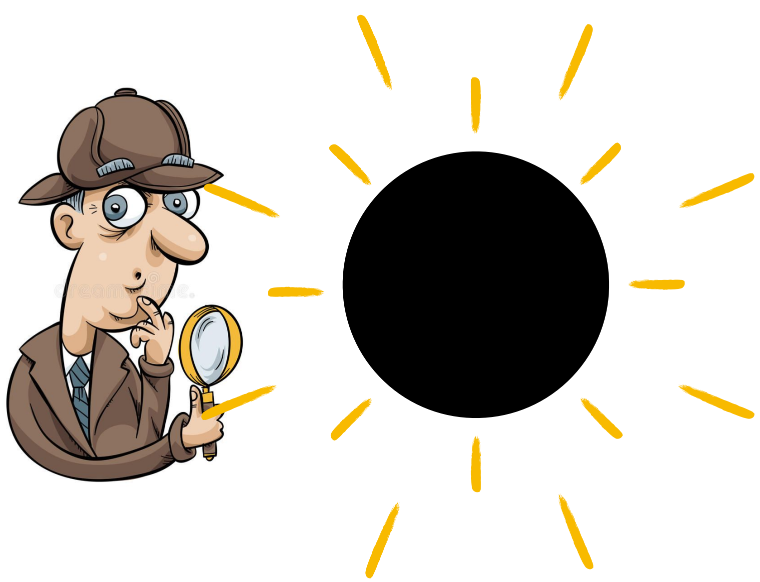
[Maldacena ’03;
Anous, Hartman, Rovai, Sonner ’16;
Saad, Shenker, Stanford ’19]



Evolution to Thermal state

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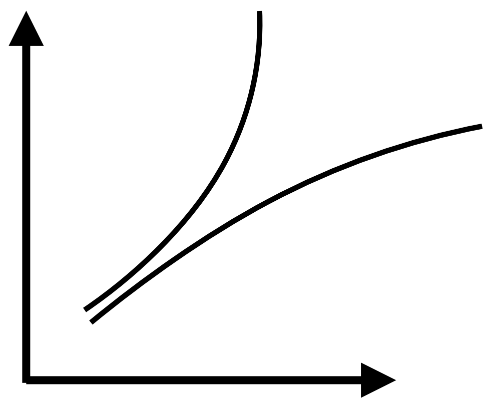
- Chaotic behaviour of the many-body systems facilitates their ergodic behaviour:



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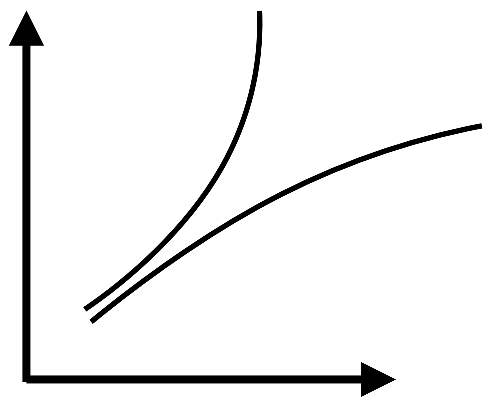

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- In quantum mechanics, one doesn't have the notion of trajectories.

  The 'distance' between the states defined as an inner product between the states remains independent of time (unitarity)

$$\langle \psi(t) \,|\, \phi(t) \rangle = \langle \psi(0) \,|\, \phi(0) \rangle$$

Quantum Ergodicity

* ω is the level spacing between eigenstates

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- Berry-Tabor/BGS conjecture:

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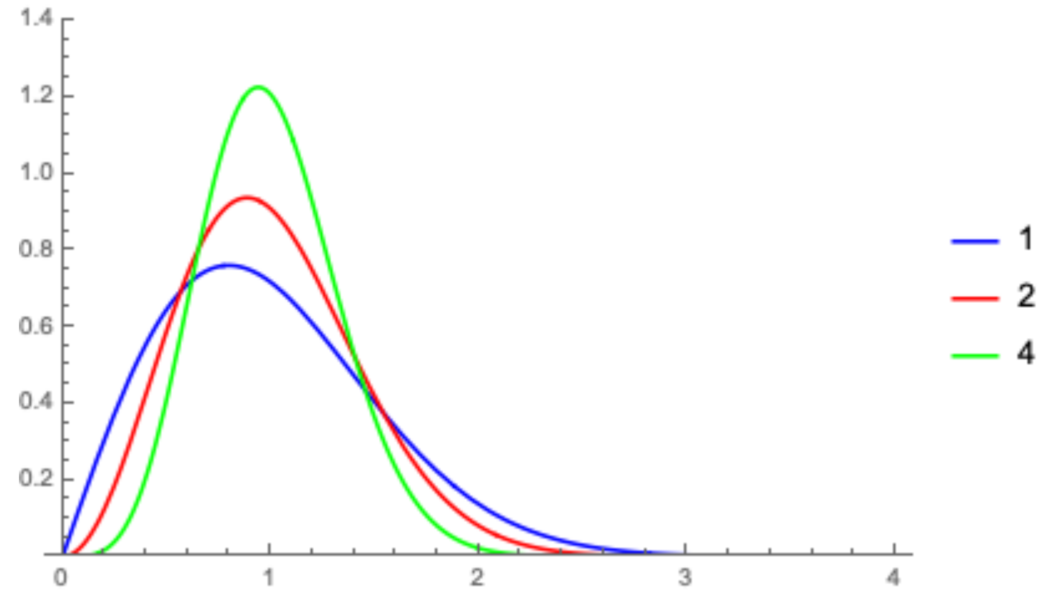
  - ✦ In Chaotic Quantum systems, *spectral statistical properties* are ‘similar’ to those of a Random Matrix Theory (RMT)

$$P^n[\omega] = A_n \omega^n \exp\left[-B_n \omega^2\right]$$



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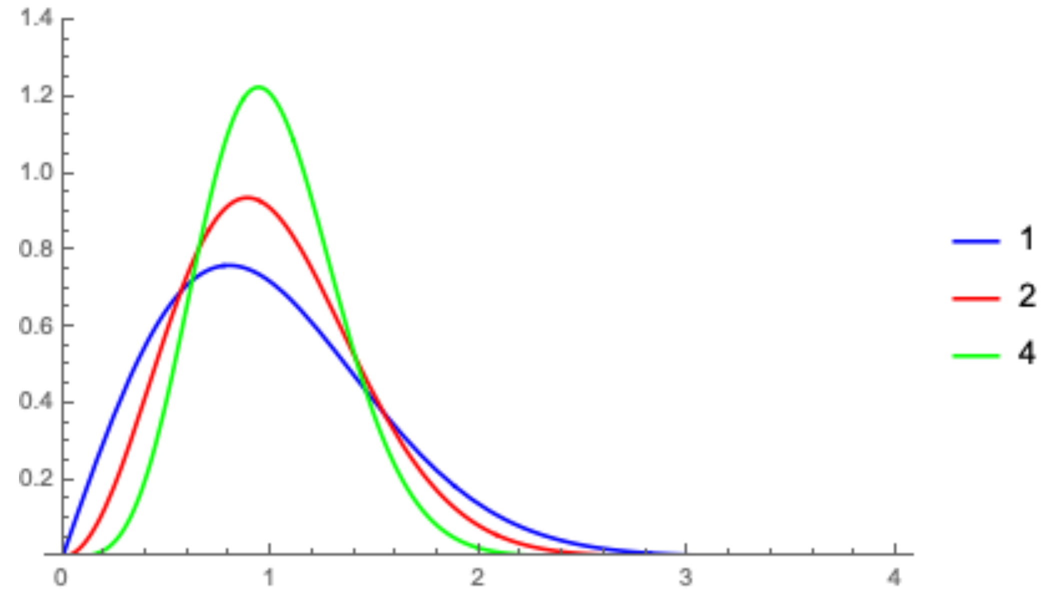

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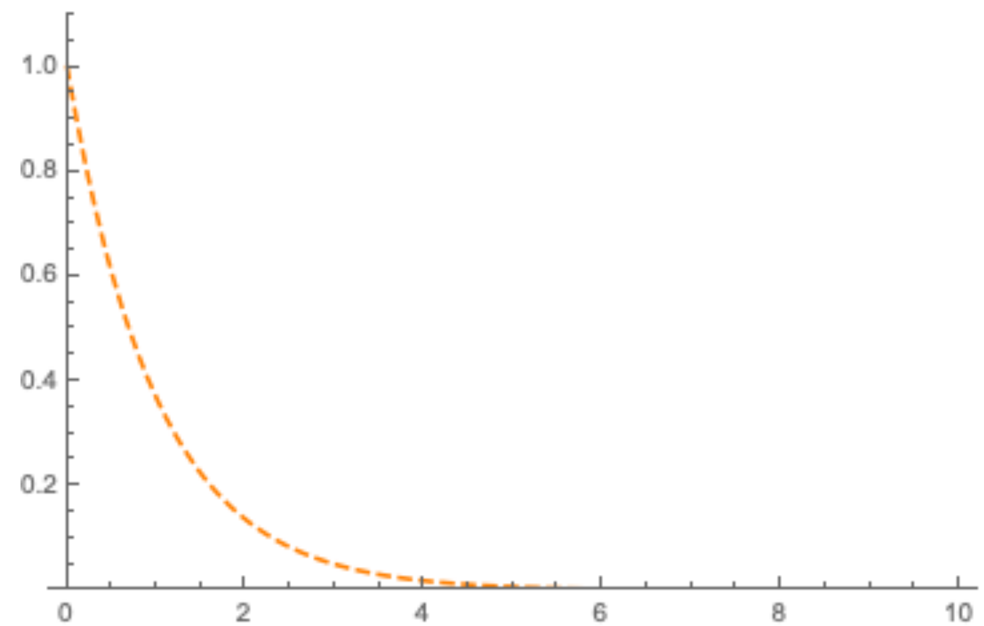

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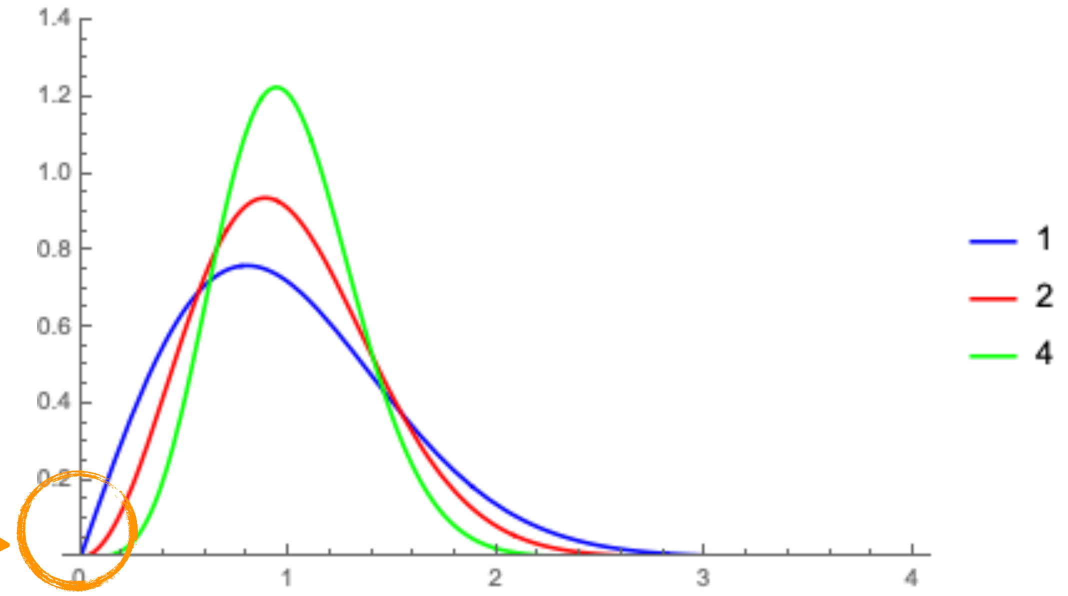
Level repulsion



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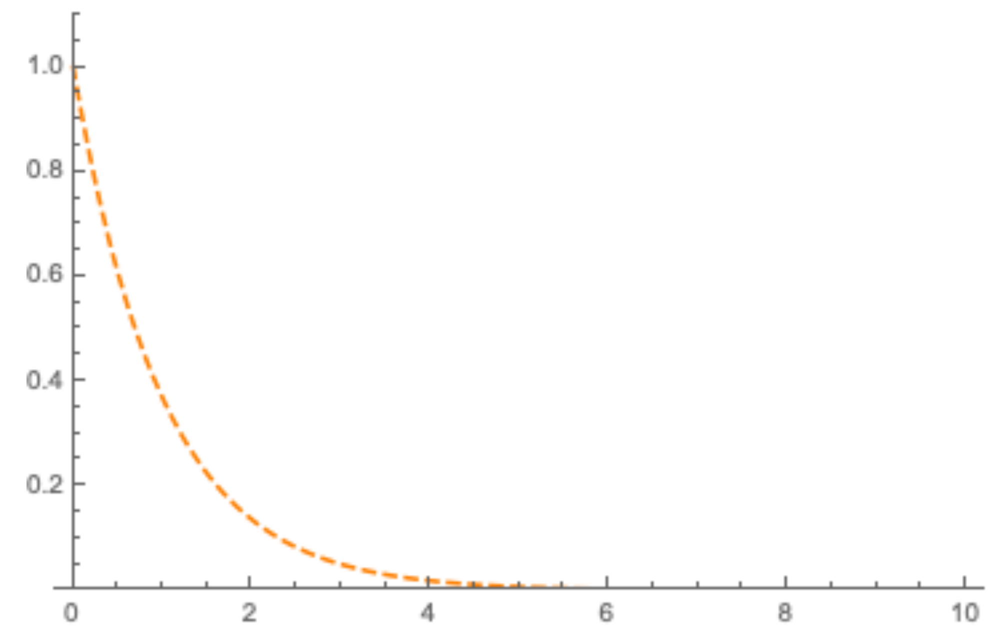

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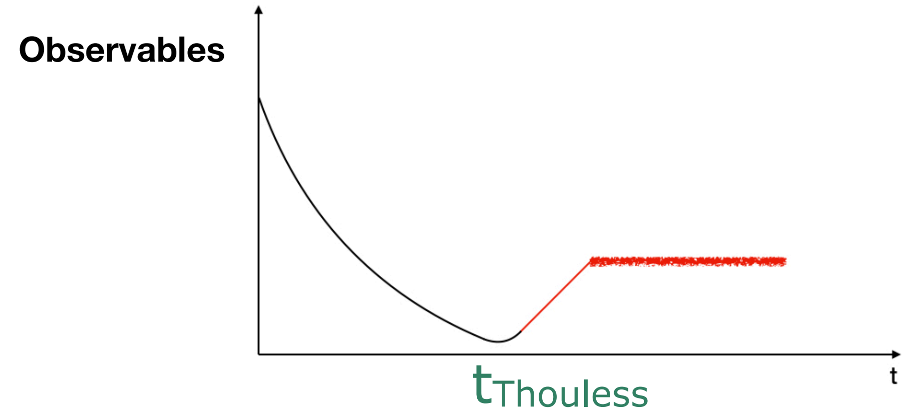

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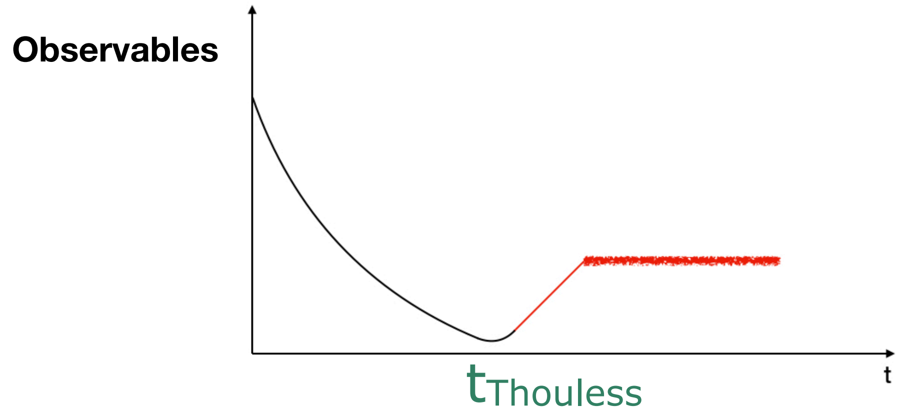

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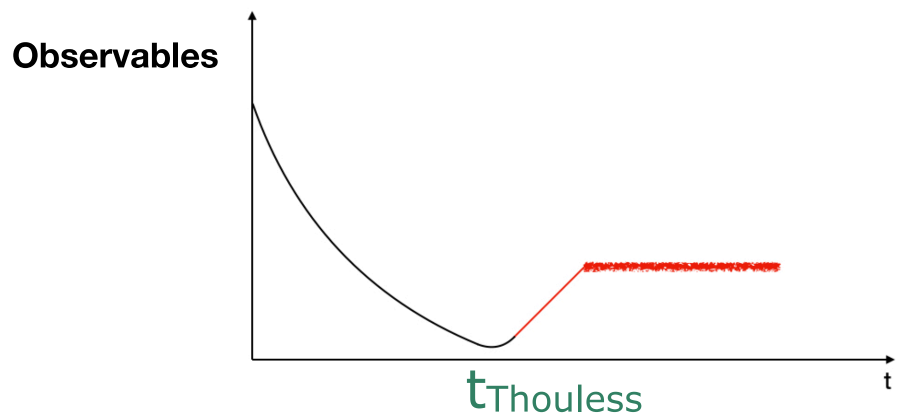

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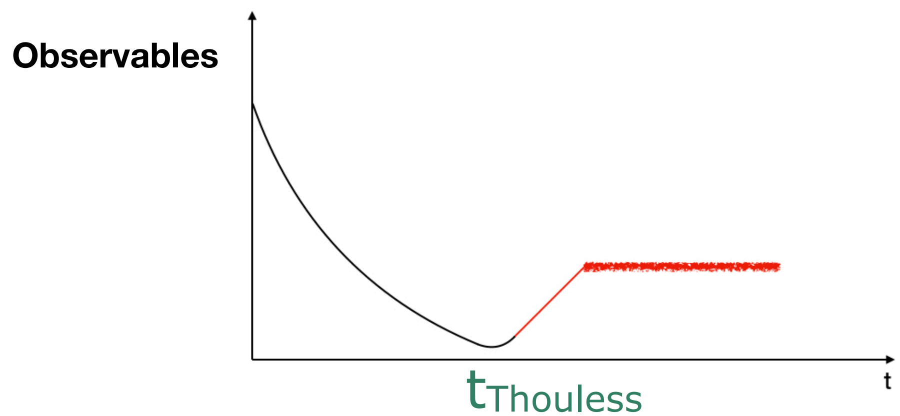

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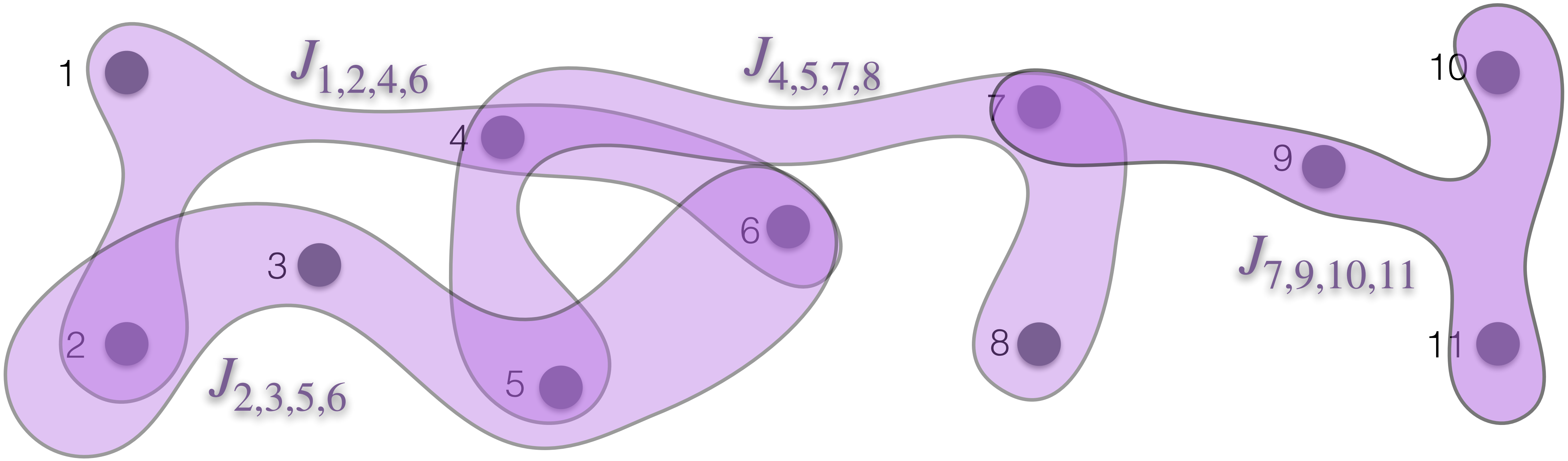

[Sachdev-Ye ’93;
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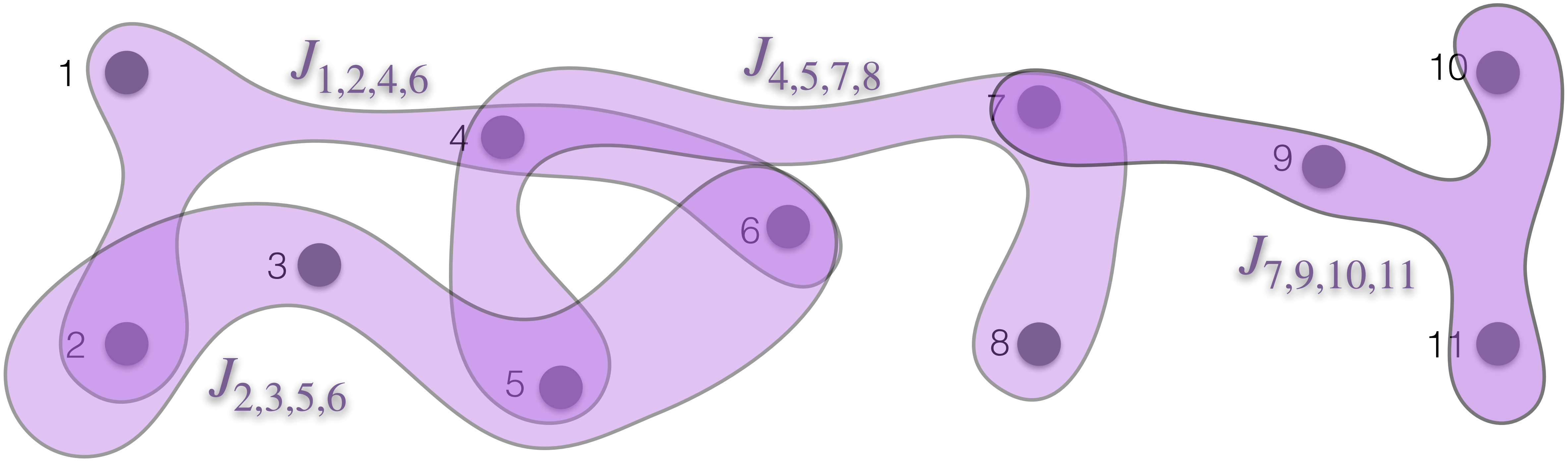

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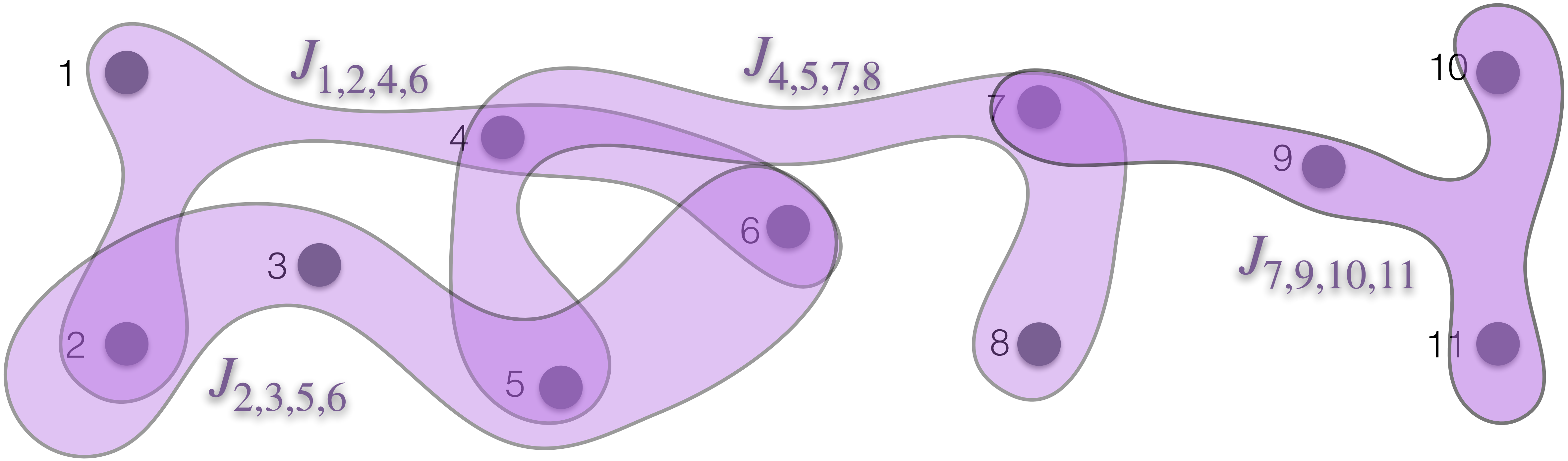

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$$H = -\sum_{1\leq i_1<i_2<\cdots<i_q\leq N} J_{i_1 i_2 \cdots i_q}\, \psi_{i_1}\psi_{i_2}\cdots\psi_{i_q}$$

where, J is chosen from a Gaussian ensemble:

$$\langle J_{i_1\cdots i_q}\rangle = 0 \qquad \langle J^2_{i\cdots i_q}\rangle = \frac{(q-1)!\, J^2}{N^3}$$

Universality of Quantum Ergodicity

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- The spectral statistics can be captured by computing the spectral resolvent,

$$R_2(\omega) \sim \left\langle \rho\left(E+\frac{\omega}{2}\right)\rho\left(E-\frac{\omega}{2}\right)\right\rangle_c \sim Re\left\langle G^+\left(E+\frac{\omega}{2}\right)G^-\left(E-\frac{\omega}{2}\right)\right\rangle$$

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  [Altland, Bagrets '17; Altland, PN, Sonner, Vielma *ongoing*]

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- The coset manifold for a specific system depends on the symmetries of the original Hamiltonian
  For the case of time-reversal non-symmetric systems the coset manifold is $U(2|2)/U(1|1) \times U(1|1)$ [Altland Zirnbauer '97]

Operators in Ergodic Limit

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- Operator correlation functions in the ergodic limit show a similar behaviour [Altland, PN, Sonner, Vielma *ongoing*]

$$\tilde{R}(E,\omega) = \sum_{\beta} |\langle \alpha | \mathcal{O} | \beta \rangle|^2 \, \delta\left(E_\alpha - E_\beta - \omega\right)$$

$$\approx \mathrm{Tr}[\mathcal{O}]\mathrm{Tr}\left[\mathcal{O}^\dagger\right] \pi\, \delta(x) + \mathrm{Tr}\left[\mathcal{O}\mathcal{O}^\dagger\right] \left(2\pi\delta(x) - \frac{\sin^2(x)}{x^2}\right)$$

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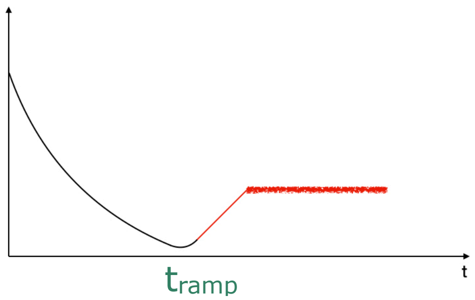

- We also study the contribution of the non-universal “Thouless modes” to the above operator correlation function.

Thermalisation in Pure states?

Thermalization in different pure states of the SYK model

state	2D picture	class	ETH	λ
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$\lvert E_{r-}\rangle$	FZZT	elliptic	✔	$2\pi T_{\mathrm{ETH}}$
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$\mathcal{O}_{\ell_H}\lvert 0\rangle$	ZZ	parabolic	✔	—

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$\mathcal{O}_{\ell_H}\|0\rangle$	ZZ	parabolic	✔	—

[PN, Sonner, Vielma 1903.00478; 1907.10061]

Summary

- ✦ We have developed a microscopic understanding of the emergence of thermal behaviour in a physical quantum mechanical system, the SYK model.

- ✦ We did this by explicitly deriving the RMT-like spectral statistics in this system.

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- E T H
  In terms of an arbitrary correlation function,

$$\langle m | \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_k | n \rangle \sim \langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_k \rangle_{mc} \, \delta_{mn} \; + \; e^{O\left(-S(\bar{E})/2\right)}$$

$$\langle m | OTOC | m \rangle \sim 1 - \frac{\#}{C} e^{\lambda_L t},$$

where, $\lambda_L \sim \dfrac{2\pi}{\beta(\bar{E})}$

[Sonner, Vielma ’17]

More General ETH

- E T H

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[Sonner, Vielma ’17]

- In a Conformal field theory, we have state-operator correspondence, $|m\rangle \leftrightarrow \mathcal{O}_m$

[Dymarsky, Lashkari, Liu ’16, ’17]

  In such a case,

$$\langle m | \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_k | n \rangle \leftrightarrow \langle \mathcal{O}_m \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_k \mathcal{O}_n \rangle \sim \langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_k \rangle_{mc} \; + \; e^{O\left(-S(\bar{E})/2\right)}$$

in particular for the 3-point function: $c_{mkn} = f_k(\overline{E})\delta_{mn} + \mathcal{O}\left[e^{-S(\overline{E})/2}\right]$

Schwarzian in SYK

[Kitaev '15; Maldacena, Stanford '16]

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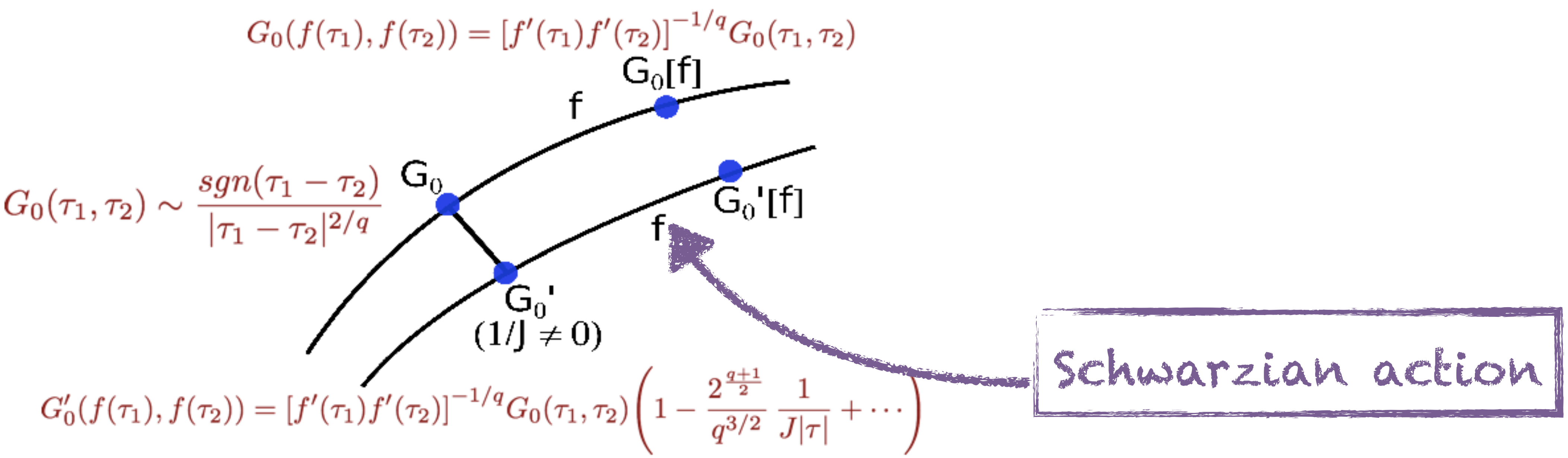
- At low-energy there is an emergent conformal symmetry, that is broken spontaneously as well as explicitly… Leading soft-mode physics



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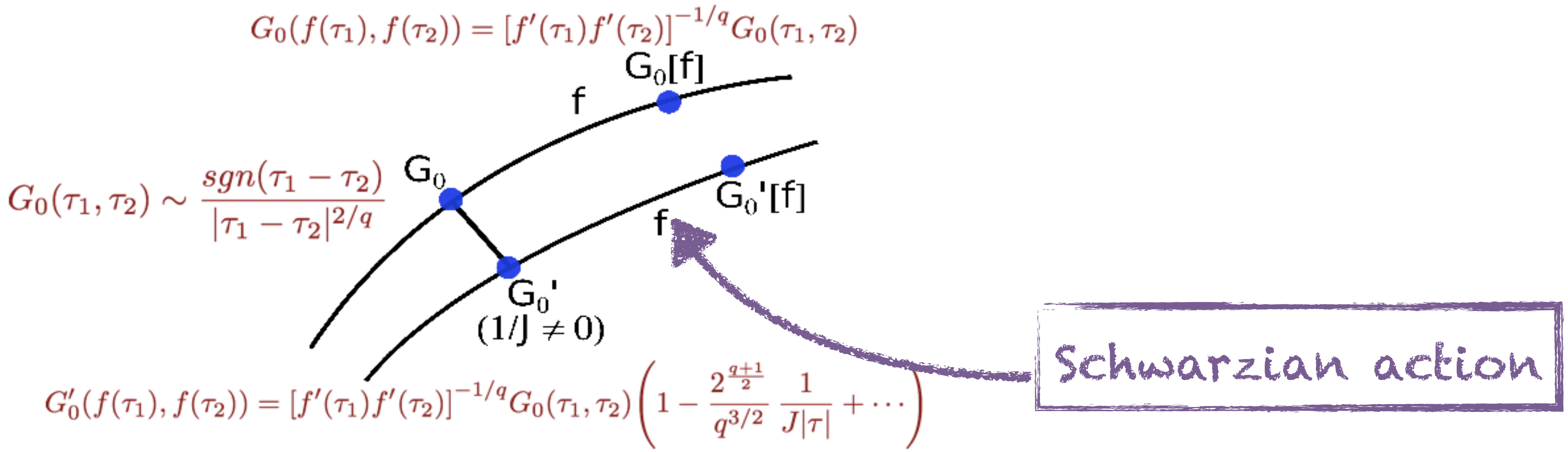

- Effective action on the 'reparametrization modes'

$$\int \frac{f(\tau)}{\mathbb{SL}(2,\mathbb{R})} \exp\left[-\frac{1}{g^2}\int d\tau\, \{f(\tau), \tau\}\right]$$

where, $$\{f(\tau), \tau\} = \frac{f'''(\tau)}{f'(\tau)} - \frac{3}{2}\left(\frac{f''(\tau)}{f'(\tau)}\right)^2 \qquad g^2 \sim \frac{\beta J}{N}$$

Spectrum

[Kitaev ’15; Polchinski, Rosenhaus ’15; Jevicki, Suzuki, Yoon ’16; Maldacena, Stanford ’16; Gross, Rosenhaus ’17]

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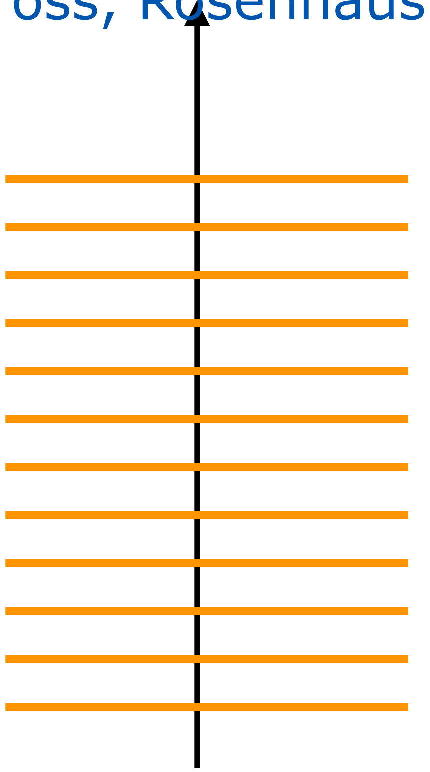


discrete tower of states

$$\mathcal{O}_n \sim \psi_i \, \partial^{2n+1} \psi_i$$

$$h_n = 2n + 1 + \epsilon_n$$

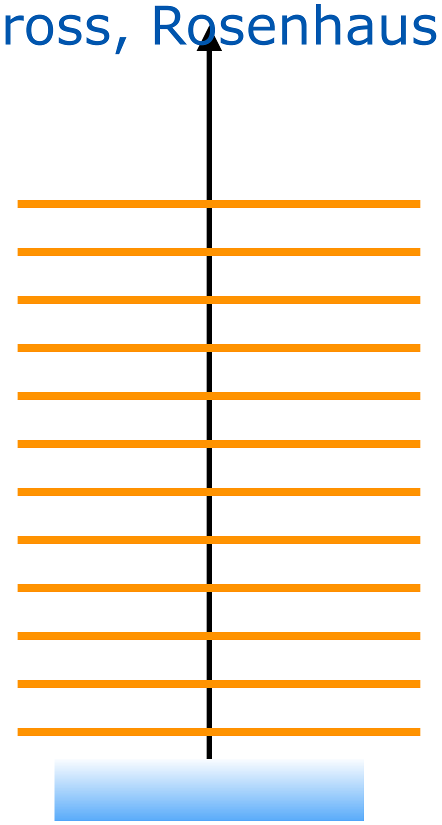
Spectrum

[Kitaev '15; Polchinski, Rosenhaus '15; Jevicki, Suzuki, Yoon '16; Maldacena, Stanford '16; Gross, Rosenhaus '17]

continuum: Schwarzian

$$S = -\frac{1}{g^2}\int d\tau \, \{f(\tau), \tau\}$$

$$f(\tau) \in \textbf{Diff}(S^1)$$



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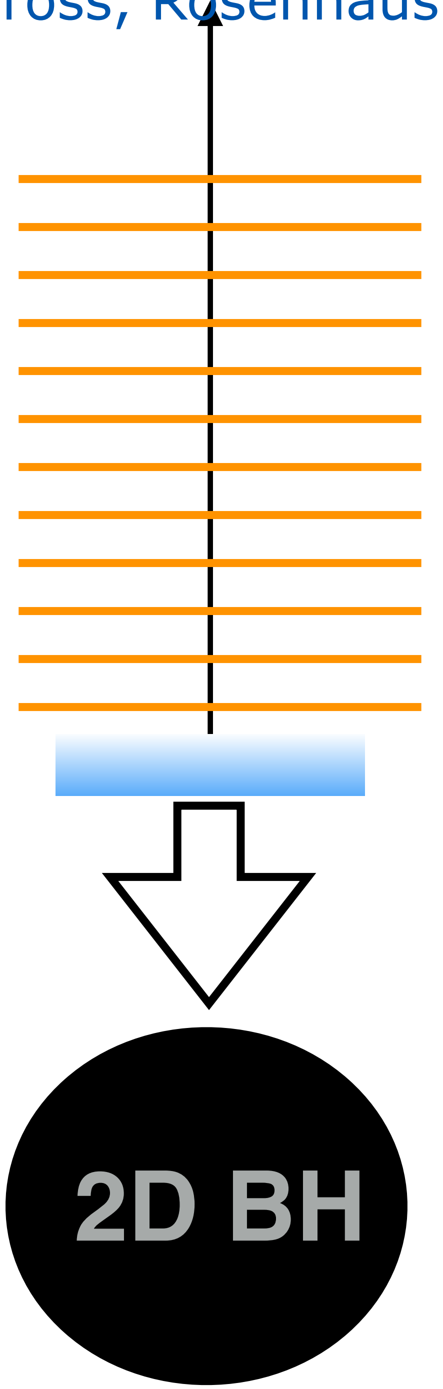

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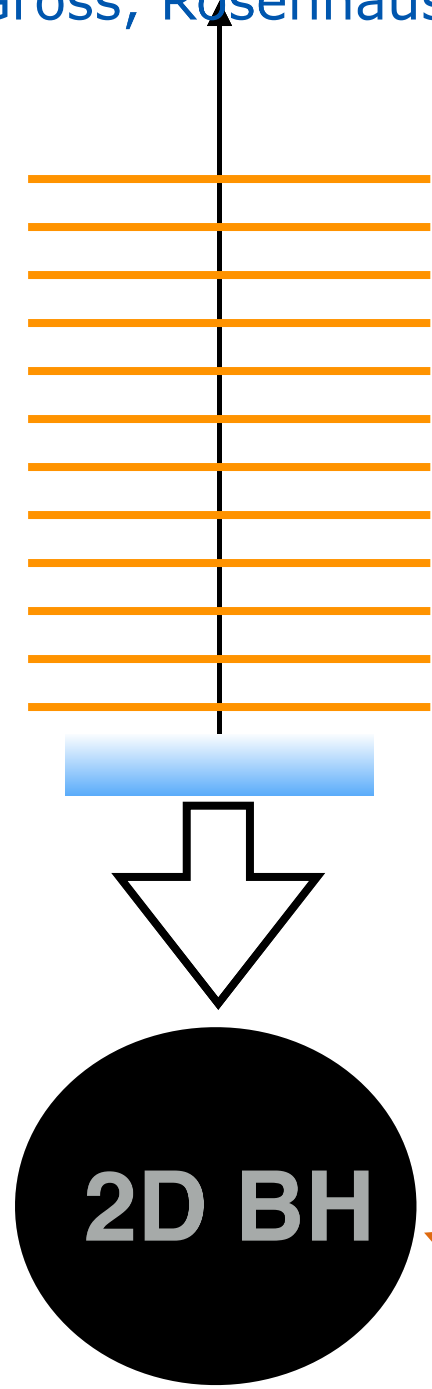

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Limit of conformal
six-pt functions
OPE coeffs.

ETH

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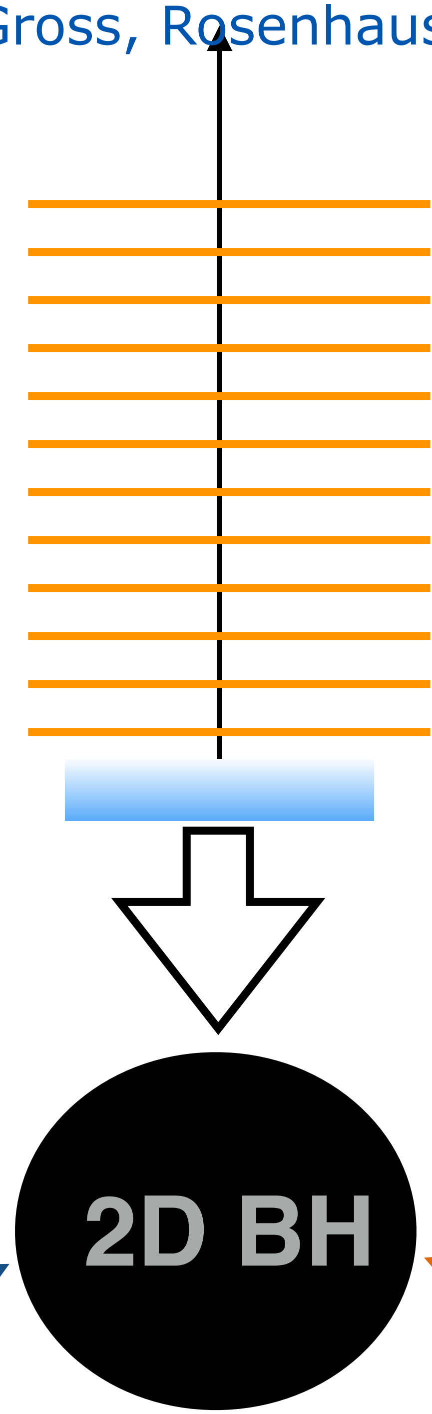

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Using `duality'
between 2D Liouville
and Schwarzian theory

ETH

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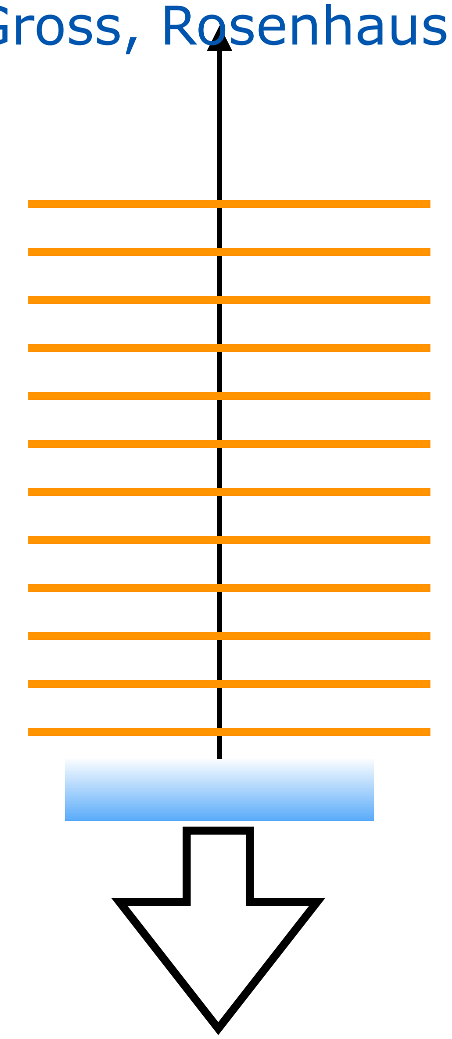

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2D BH

Limit of conformal
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ETH

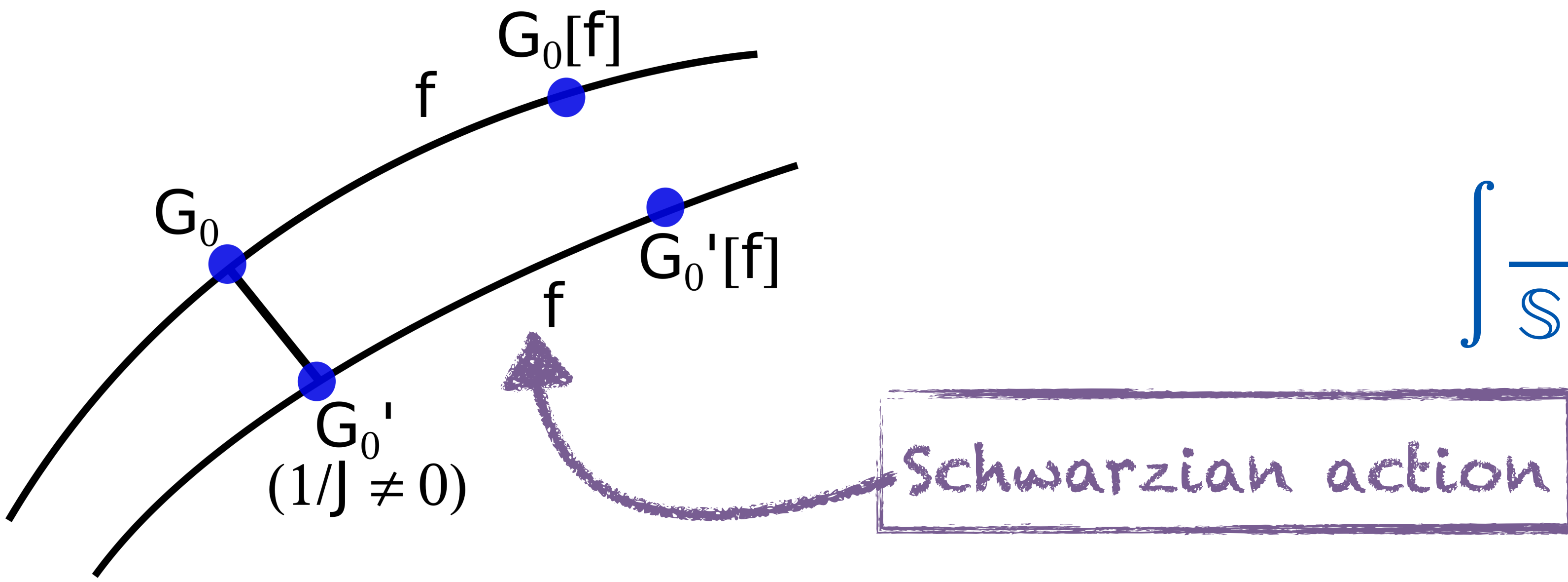
ETH

[PN, Julian Sonner & Manuel Vielma]

The Schwarzian theory

The Schwarzian theory

- For large but finite values of SYK coupling J, the conformal symmetry is explicitly broken



$$\int \frac{f(\tau)}{\mathbb{SL}(2,\mathbb{R})} \exp\left[-\frac{1}{g^2}\int d\tau \left\{f(\tau), \tau\right\}\right]$$

$$g^2 \sim \frac{\beta J}{N}$$

The Schwarzian theory

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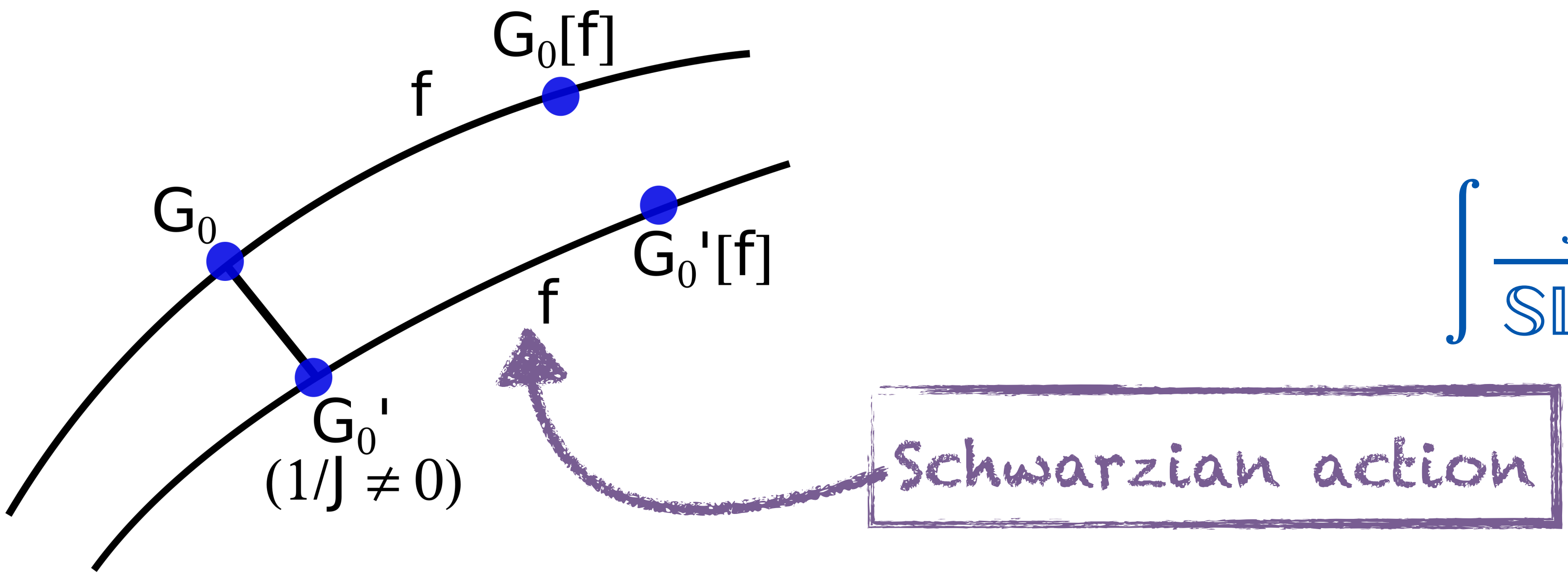

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$$g^2 \sim \frac{\beta J}{N}$$

- $f(\tau)$ are the pseudo-Goldstone modes

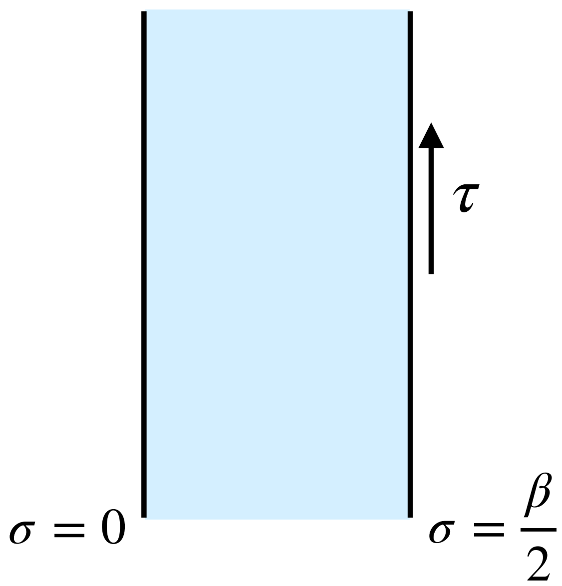
The leading contribution to the physical observables is due to exchange of these modes

$$\langle \cdot \rangle = \int \frac{f(\tau)}{\mathbb{SL}(2,\mathbb{R})} \exp\left[-\frac{1}{g^2}\int d\tau \left\{f(\tau),\tau\right\}\right] \left(\cdot \right)$$

Schwarzian theory from Liouville theory

Schwarzian theory from Liouville theory

- Liouville theory on an open string,



$$\mathscr{H} = \frac{1}{16\pi b^2}\left[\pi_\varphi^2 + \partial_\sigma \varphi^2 + 2e^{\varphi} - \varphi_{\sigma\sigma}\right]$$

$$c = 1 + 6\left(\frac{1}{b} + b\right)^2$$

Schwarzian theory from Liouville theory

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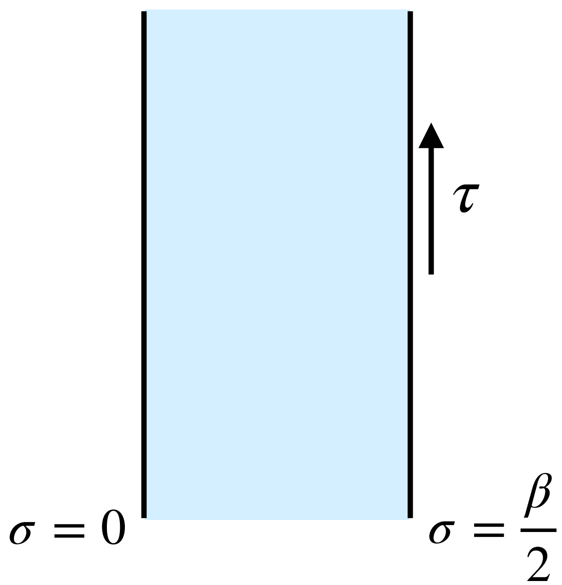

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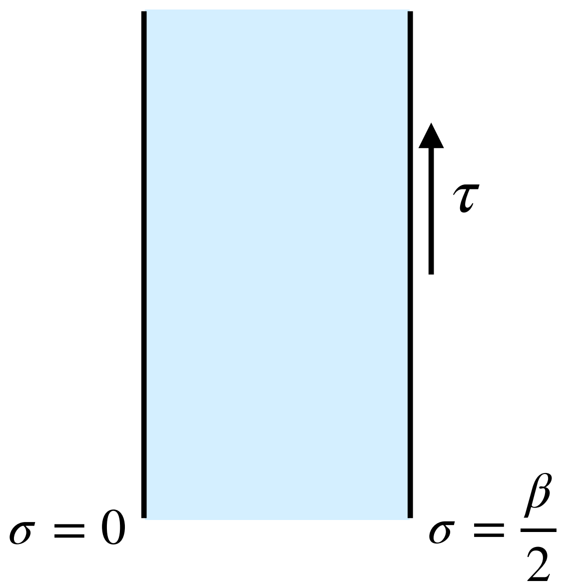
- Different boundary conditions consistent with the Conformal symmetry:

Schwarzian theory from Liouville theory

[Mertens Turiaci Verlinde '17; Lam Mertens Turiaci Verlinde '18]

- Liouville theory on an open string,



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- Different boundary conditions consistent with the Conformal symmetry:

[PN Sonner Vielma '19]

Dirichlet: $\varphi = \infty$

Neumann: **or,** $\partial_\sigma \varphi = -re^{\varphi/2}$ **on the boundaries**

- Action with the boundary term:

$$\int \mathscr{D}\varphi \mathscr{D}\pi e^{\int d\tau d\sigma\left[\frac{\pi\dot{\varphi}}{8\pi b^2} - \mathscr{H}\right] + S_{bdy}}$$

$$S_{bdy} = -\frac{r}{4\pi b^2}\int d\tau\, e^{\frac{1}{2}\varphi}$$

Liouville -> Schwarzian

- With a judicial choice of field variables along with the classical limit, $b \to 0 \Rightarrow$

$$S_{\text{Liou}} \quad \to \quad C \int_{\beta/2}^{\beta/2} d\sigma \left[\{f(\sigma), \sigma\} + \frac{2\pi^2\theta^2}{\beta^2} f'(\sigma)^2 \right] - \frac{4\pi C}{\beta} \frac{f'(\beta/2)}{\tan(\pi\theta)}$$

$$C = \frac{a}{4\pi b^2}$$

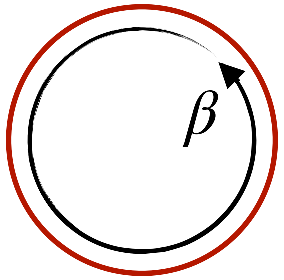
size of the temporal direction

$$r = \sqrt{2}\cos(\pi\theta)$$

Liouville -> Schwarzian

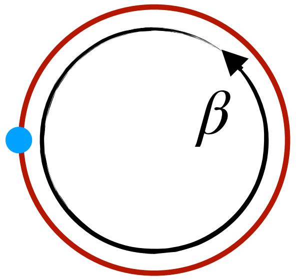
- Dirichlet-Dirichlet:

$$\int \frac{\mathcal{D}f}{\mathrm{SL}(2,\mathbb{R})} \exp\left[C \int_{-\beta/2}^{\beta/2} d\sigma \left(\{f(\sigma),\sigma\} + \frac{2\pi}{\beta^2} f'(\sigma)^2 \right) \right]$$



- Dirichlet-Neumann:

$$\int \frac{\mathcal{D}f}{\mathrm{U}(1)} \exp\left[C \int_{-\beta/2}^{\beta/2} d\sigma \left(\{f(\sigma),\sigma\} + \frac{2\pi\theta^2}{\beta^2} f'(\sigma)^2 \right) - \frac{4\pi C}{\beta \tan(\pi\theta)} f'\left(\frac{\beta}{2}\right) \right]$$



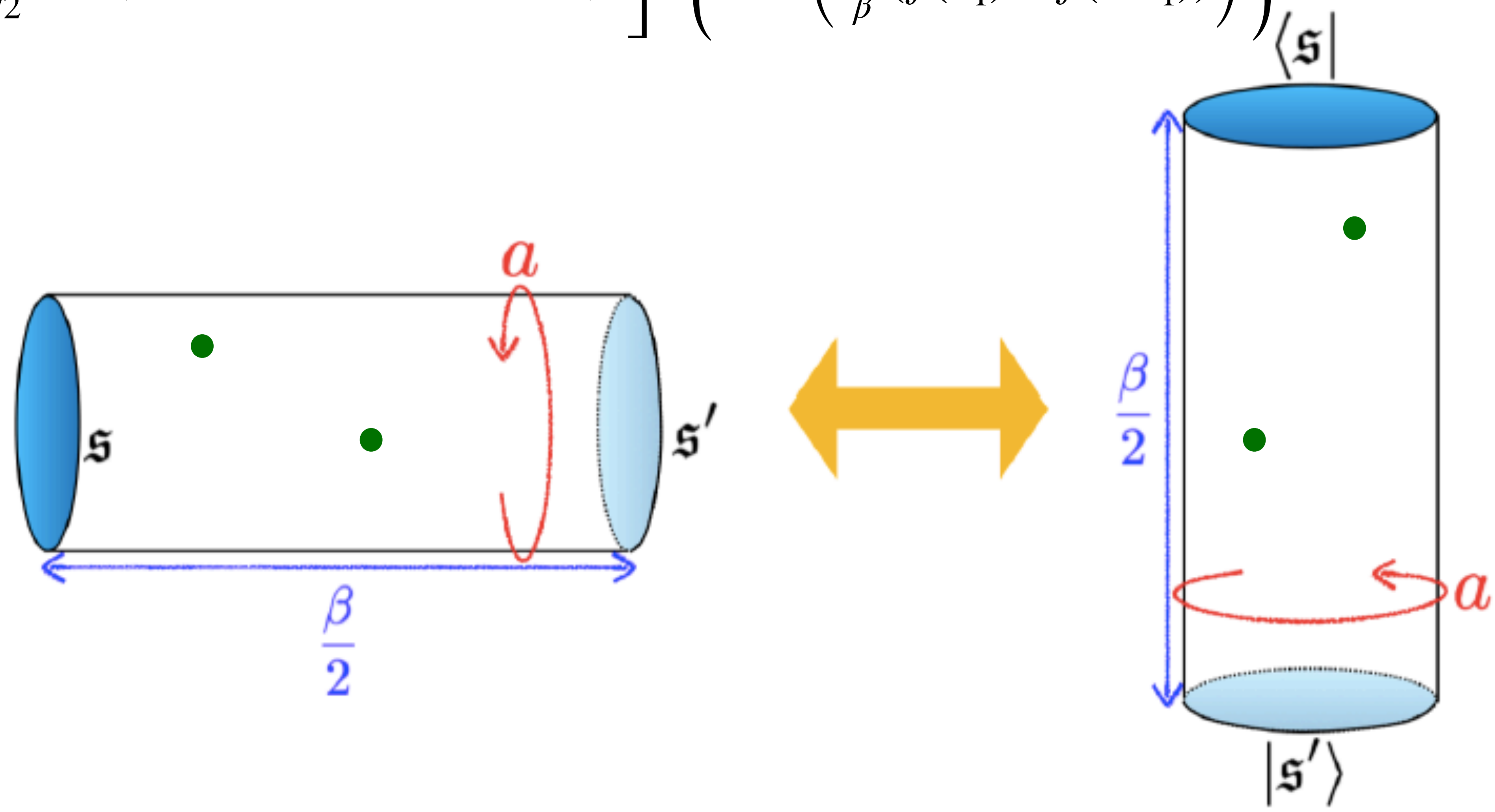
Liouville -> Schwarzian

Moreover, the vertex operators in the Liouville theory reduce to bilocal operators in the Schwarzian theory:

$$e^{2\ell\varphi(z,\bar{z})} \to \left[\frac{f'(\sigma)f'(-\sigma)}{\sin^2\left(\frac{\pi\theta}{\beta}(f(\sigma) - f(-\sigma))\right)} \right]^{2\ell} =: \mathbb{O}_\ell(\sigma, -\sigma)$$

Liouville -> Schwarzian

$$\int \frac{\mathscr{D}f}{G} \exp\left[C \int_{-\beta/2}^{\beta/2} d\sigma \left(\{f(\sigma), \sigma\} + \frac{2\pi\theta^2}{\beta^2} f'(\sigma)^2 \right)\right] \left(\frac{f'(\sigma_1) f'(-\sigma_1)}{\sin^2\left(\frac{\pi\theta}{\beta}(f(\sigma_1) - f(-\sigma_1))\right)} \right)^{2\ell_1} \cdots$$



$$\langle s| = \int dP\ \Psi_s(P)\ \langle\langle P|$$

Ishibashi states: $\langle\langle P| = \langle \nu_P| \left(1 + \frac{L_1 \bar{L}_1}{\Delta_P} + \ldots \right),\qquad |\nu_P\rangle = e^{\frac{1}{4b}(\frac{Q}{2}+iP)\varphi}$