Deformation quantization, Batalin-Vilkovisky formalism and index

Villa Battelle (Université de Genève) – March 18-21, 2019

Colloquium Talks

Damien Calaque (University of Montpellier)

Title :

Weyl dimension formula for the deformation quantization of coadjoint orbits

Abstract :

This mini-corse will be devoted to a proof of an analogue of the Weyl dimension formula for the Aleeksev-Lachowska star-product on coadjoint orbits.

The first lecture will be devoted to the basics of deformation quantization, and to the construction of the Alekseev-Lachowska star product on semi-simple regular coadjoint orbits.

In the second lecture, we will explain several general results (mainly due to Fedosov, but also Nest-Tsygan, Deligne, Karabegov and Bressler-Donin) about deformation quantization of symplectic manifolds: existence, classification, trace formula, ...

The last lecture will be devoted to the proof of an analogue of Weyl dimension formula. If time permits, we will provide a few perspectives and open questions.

Alberto Cattaneo (University of Zurich)

Title :

Poisson sigma model by cut and paste

Abstract :

In these talks, I will start recalling the classical theory of the Poisson sigma model (PSM) and its relation to the integration of Poisson manifolds. I will then describe its quantum theory and its relation to deformation quantization.

Next I will describe the globalization issue and describe how it can be incorporated into the PSM via the BV formalism.

Finally, I will present an application of the BV-BFV formalism: the quantization of the relational symplectic groupoid and the lift (up to homotopy) of deformation quantization to the path space.

Giovanni Felder (ETH Zurich)

Title :

Hochschild cohomology of polynomial differential operators and applications

Abstract :

I will review a construction of explicit sp(2n)-basic Hochschild cocycles of the algebra of differential operators with polynomial coefficients in n variables, obtained with B. Feigin and B. Shoikhet. Such cocycles are building blocks for trace densities and index theorems in deformation quantization of symplectic manifolds. Another application is a local formula, first conjectured by B. Feigin, A. Losev and B. Shoikhet, for the Lefshetz number of holomorphic differential operators on complex manifolds, generalizing the Riemann-Roch-Hirzebruch theorem. Time permitting, I will also discuss the origin of the formula in Kontsevich's formality theory (Shoikhet's proof of Tsygan's conjecture) and an extension to cyclic cohomology due to Willwacher.

Si Li (Tsinghua University)

Title :

Effective Batalin-Vilkovisky quantization and geometric applications

Abstract :

We discuss the homological method of Batalin-Vilkovisky (BV) quantization in gauge theories. We introduce basics of infinite dimensional technique of Costello's homotopic renormalization method, and explain its geometric context in quantum field examples. We present its applications to several geometric *problems, including algebraic index theorem in 1d, chiral index theorem in 2d, open-closed topological string field theory in mirror symmetry and large N duality/twisted holography.*

Pavel Mnev (University of Notre Dame)

Title :
BF theory on cell complexes
Abstract :
I will discuss the topological non-abelian BF theory on a cell complex, with a finite-dimensional space of fields modelled on cell cochains and satisfying the Batalin-Vilkovisky quantum master equation. Topological invariants are produced via finite-dimensional integrals (instead of the path integral); the theory is well-behaved with respect to cellular aggregations (inverses of subdivisions) and, more generally, with respect to simple-homotopy equivalence. In the case of a manifold endowed with a triangulation (or, more generally, a prismatic decomposition), the theory is constructed as effective theory on Whitney forms. I will also explain in which sense this model satisfies an Atiyah-Segal-like gluing axiom. This is in part a joint work with Alberto S. Cattaneo and Nicolai Reshetikhin.

Sylvie Paycha (University of Potsdam)

Talk 1
Title :
Are locality and renormalisation reconcilable?
Abstract :
According to the principle of locality in physics, events taking place at different locations should behave independently, a feature expected to be reflected in the measurements. The latter are confronted with theoretic predictions which use renormalisation techniques in order to deal with divergences from which one wants to derive finite quantities. The purpose of this talk is to confront locality and renormalisation. Sophisticated (co)algebraic methods developed by physicists enable to keep track of locality while renormalising. They mostly use a univariate regularisation scheme such as dimensional regularisation. We shall present an alternative multivariate approach to renormalisation which encodes locality as an underlying algebraic principle. We shall apply it to various situations involving renormalisation, such as divergent multizeta functions and their generalisations, namely discrete sums on cones and discrete sums associated with trees.

This is based on joint work with Pierre Clavier, Li Guo and Bin Zhang

Talk 2
Title :
A geometric perspective on Hairer's regularity structure
Abstract :
We use groupoids to describe a geometric framework which can host a generalisation of Hairer's regularity structures to manifolds. The latter offer an algebraic device in order to transform a singular stochastic differential equation into a fixed point problem, by means of an ad hoc ``Taylor expansion'' of the solutions at any point in space-time and a ``re-expansion map'' which relates the values at different points. To give a geometric interpretation of Hairer's re-expansion map we define direct connections on gauge groupoids; these generalise Teleman's direct connections on morphism bundles. The re-expansion map can therefore be viewed as a (local) ``gaugeoid field'', the groupoid counterpart of a (local) gauge field. In the case of Riemannian manifolds without boundary, we compare our definition of a polynomial regularity structure with the one given by Driver, Diehl and Dahlquist.

This talk is based on joint work with S. Azzali, Y. Boutaïb and A. Frabetti.