SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

SwissMAP Perspectives

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SWISSMAP RESEARCH STATION

The SwissMAP Research Station (SRS) has become a valuable resource for our scientific community. Since its founding in 2021, it has hosted over 60 international conferences, workshops, and educational programs covering various topics. In the past year, it attracted over 850 researchers and scientists from a wide range of disciplines worldwide.

Beyond its borders, the Research Station has recently become an institutional member of the European Mathematical Society and European Research Centres on Mathematics (ERCOM). Additionally, it now collaborates with G-Research, a quantitative finance research and technology company.

News, events & programs

MAPSS

Over the years, SwissMAP has allocated significant resources to master's programs and has developed extensive expertise in this area. The newly launched MAPSS (Mathematical Physics Summer Schools) initiative, designed for master's students, reflects SwissMAP's commitment to nurturing the next generation of mathematics and physics scholars.

The first edition of MAPSS was open to students from Swiss institutions and took place this year from July 14 to 19 at the SwissMAP Research Station. There were 41 participants from six different institutions across Switzerland.

The program featured 7 introductory mini-courses on the following topics:

- Symplectic geometry
- Symplectic reduction (and its applications in physics)
- Differential geometry
- General relativity
- Algebraic topology
- Supergeometry (and its applications in physics)
- Singular ODEs

The second edition of MAPSS will take place in August 2025.

SRS²

The program aims to foster collaboration, innovation, and interdisciplinary research. By working in small groups, researchers are able to leverage each other's expertise and knowledge, leading to more comprehensive and well-rounded research outcomes.
Between May and August 2024, 5 groups of 4 to 6 researchers met for one week at the SRS². Francesco Riva (UNIGE) commented: "*The week has completely exceeded all our expectations: the format was just perfect and we think we made a lot of progress in different interesting directions. We had 4 days with one talk in the morning and then discussed hamiltonian truncation for the whole day... we have designed an impressive list of concrete projects for the near future that we are all excited to start.*"

SwissMAP Junior Researcher Prize

Applications and nominations for the first SwissMAP Junior Researcher Prize sponsored by G-Research are now open. This new broader prize will replace the SwissMAP Innovator Prize.

Application deadline:
September 30, 2024

The 2024 Prize ceremony will take place at the SRS in Les Diablerets on Wednesday, 8th January 2025, during the annual Winter School in Mathematical Physics.

Other News

We recently obtained outdoor blackboards to take advantage of the beautiful surroundings.

Visit our website

- Subscribe to our SRS mailing list and keep up to date on the call for proposals
- Discover our extensive video library on our website
- Follow us on Instagram

https://swissmaprs.ch/

OUTREACH

Additionally, the SRS promotes the development of the next generation of researchers by organizing various educational outreach programs and activities for school students, young scientists, and the general public.

Les Marmottes

Last year saw the successful launch of "Les Marmottes – Filles et Maths", a workshop aimed at middle school-aged girls. The second edition extended to 5 days of workshop and invited girls from Vaud to participate.

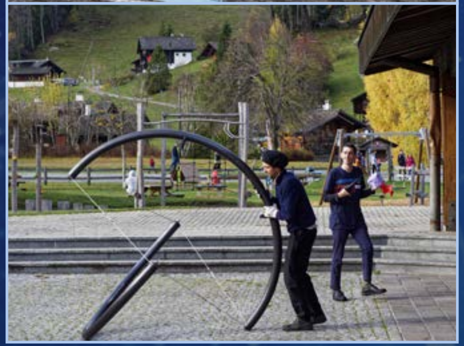
Once again, the program received press coverage from RTS, this time with the radio interviewing some of the girls and organizers.

MATRIX × IMAGINARY 2024 & Math'émerveille

This year, the SRS hosted the international conference MATRIX × IMAGINARY 2024 on the future of mathematics engagement from 31st August to 4th September. The conference included participants from mathematics museums, math outreach organizations, and researchers involved in communicating mathematics. There were a total of over 120 participants from 12 ifferent countries.

Like the previous SRS outreach conference, "Let's Talk About Outreach", in 2022, the Math'émerveille Math Festival for the General Public followed this year's MATRIX × IMAGINARY. On this occasion, the festival also celebrated the inauguration of Genève Évasions Mathématiques (G·EM), the University of Geneva's new structure dedicated to scientific communication in mathematics.

The festival took place at the Musée d'Histoire des Sciences in Geneva and featured a wide range of interactive booths, a maths-related theatrical performance, and an Alice in Wonderland-themed treasure hunt, which had three distinct levels of difficulty to accommodate all participants.



Genève Évasions Mathématiques (G·EM)

The University of Geneva's new strucutre dedicated to scientific communication in mathematics, directed by Hugo Duminil Copin & Elise Raphael aims to present mathematics in a way that promotes inclusion and accessibility for all audiences.

Throughout the year, G·EM will offer a diverse range of activities, from public lectures by renowned researchers on cutting-edge research, to exhibitions and events designed to spark curiosity and interest in mathematics. There will be age-appropriate activities for schoolchildren, all based on extensive experience in popularizing mathematics. This initiative is designed to meet the growing demand for mathematical mediation.

Visit the website to discover the wide range of activities and subscribe to the mailing list to stay updated.

https://www.unige.ch/math/GEM/

2024 EXHIBITIONS

Perspectives: the mathematical universe of M.C. Escher

Between February 19th and April 26th, this exhibition, curated by our Science Officer Elise Raphael and Sandie Moody (UNIGE), delved into the recurring mathematical themes in the art of Dutch artist M.C. Escher (1898-1972). The exhibition showcased the intriguing harmony between artistic creativity and mathematical rigor.

Over the 56 days the exhibition was open, it hosted 20 school visits with students of various ages. Additionally, 20 adult group visits were organized for specific groups, including UNIGE alumni and Marmottes participants. In total, the exhibition attracted an impressive 3225 visitors, setting a new attendance record for UNIGE's exhibition hall.

The exhibition was also displayed in May at EPFL during the finals of the Swiss Federation of Mathematical Games and will be hosted by the University of Neuchatel and the HES-SO Sion during the fall and winter respectively.

Randomness in **Action**

On the bundesbank webpage, one can still read the description for the front side of the 10 Deutsche note: Carl Friedrich Gauss (1777-1855), mathematician, astronomer, geodesist and physicist. In the background, buildings from historic Göttingen.
In fact, the note also features the bell-shaped Gauss curve, which serves as the density function for the probability distribution of a realvalued standard Gaussian random variable.

The universal Central Limit Theorems states that for any stationary sequence of mean zero independent random variables $\{X_n\}$ with variance $1, \frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i \to N(0,1)$ converges as $n \to \infty$ in the sense of their probability distributions.

Donsker's Invariance Principle then allows one to conclude the probability distribution of $W^n(t) := \frac{1}{\sqrt{n}}\sum_{i=1}^{[nt]} X_i$, as random variables with values in the space of continuous paths from [0, 1] to $\mathbb{R}$, converges to a standard Brownian motion $(W_t : t \in [0,1])$, whose probability distribution is the Wiener measure. This elementary fact secures the position of Brownian motion as a fundamental object in stochastic analysis. A typical sample of a Brownian motion can be seen in figure 1.

A fundamental property of a Brownian motion, W_t, is the de-correlation of its distributional derivative. A 'white noise' is the 'derivative' of a Brownian motion: $\xi(t) =$ "$\frac{d}{dt}W(t)$", such independence is at the heart of Itô calculus, which rests on the all important Itô isometry:

$$\mathbb{E}\Big(\int_0^t f(s)\dot{\xi}(s)ds\Big)^2 = \mathbb{E}\int_0^t\int_0^t f(s)f(r)\mathbb{E}[\xi(s)\xi(r)]dsdr = \int_0^t f^2(s)ds.$$

Auto correlated Noise. Yet, in time series data, long range dependence (LRD) and autocorrelations over a large span are often observed. In Egypt, the river Nile overflows, leaving behind rich fertile silt for agriculture in the surrounding areas. Inadequate inundation results in only a small area of cropland would be covered with the life-giving silt, famine follows; too much flood means destruction. During the time of the pharaohs, Nilometers were used to forecast the water level in order to compute the levy of taxes (figure 2).

The hydrologist Harold Hurst was recruited to head the Physics department in Egypt, he worked there from 1906-1968. Based on the series data on the long-term storage capacity of reservoirs, he observed the presence of long-range dependence in the fluctuations of the water level in the Nile River. This "Hurst parameter" was studied and modelled by Mandelbrot and Van Ness using fractional Brownian motions. The Hurst parameter for the Nile data is thought to be *H* = 0.72. Today, climatologists actually believe that precipitation in the atmosphere exhibits also long range dependence, which is not surprising given that atmospheric moisture is an indicator to rain.

Fractional Brownian motion (fBM). Seeking a process with stationary increments and selfsimilarity, the latter means that there exists a and *H* such that $X(at) = a^H X(t)$, which naturally leads to Hermite processes. Among those the Gaussian varieties are fractional

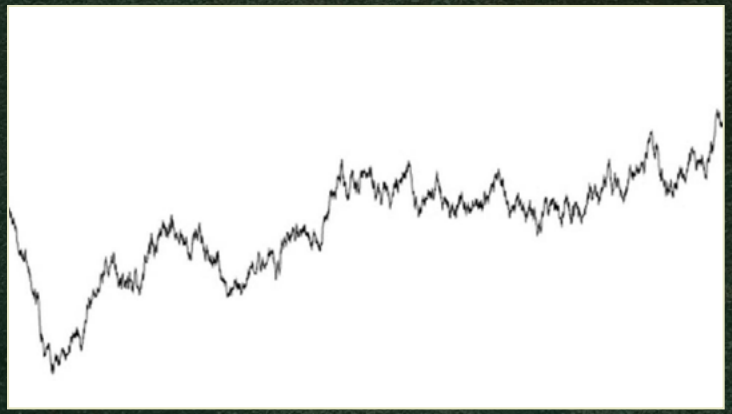


Figure 1: The Wiener measure assigns a measurement to any Borel subset of continuous paths.

Figure 2: The overflowing Nile and the Nilometers used to forecast the water level.

Brownian motions. Self-similarity emerges from scaling limit procedure. A fractional Brownian motion is a mean zero Gaussian process with covariance $\mathbb{E}(B_tB_s) = \frac{1}{2}\sigma^2(t^{2H} + s^{2H} - |t-s|^{2H})^2$. It has self-similarity exponent $H \in (0,1)$, also known as the Hurst parameter.

The correlation of increments of length 1, with a distance *n* apart, does not vanish for any *n*. This is given by $\rho(n) = \mathbb{E}[B_1(B_{n+1} - B_n)] \sim H(2H-1)\sigma^2\, n^{2H-2}$. This long-range dependence exhibits quite different properties compared to compactly supported correlations, the latter is known to dissipate through time in an evolution equation, while long-range dependence tends to persist over time. This latter was also observed in the fluctuation theory for the KPZ equation with long range spatial correlation. In the context of stochastic partial differential equations (SPDE), Gaussian noise has traditionally been assumed to be uncorrelated in time and space. However, single source randomness suggests the need for other modeling approaches. Compactly supported spatial correlation is typically obtained through a mollification procedure.

Fractional Brownian motions are the unique class of Gaussian process among those self-similar stationary increment processes. With a bit of work, one can also make sense of the Poisson equation with a white noise right-hand side, and define the fBM by: $(-\Delta)^{\frac{H}{2}+\frac{1}{4}}B_t^H = c\eta$ where η is the white noise and Δ is the Laplacian.

While embracing long range dependence there are plenty of mathematical challenges to overcome. There is now a theory of integration and stochastic differential equations driven by fractional Brownian motions, offering outlooks. However the theory is unprepared for certain problems in stochastic dynamical system problems which we intend to study.

Multi-scale problems. Multi-scale problem is concerned with interacting quantities evolving at different time scales. There is an old saying that one dog year equals seven human years. There are other examples such as interaction between virus and animals, or the evolution of celestial orbits versus that of compared to that of a human life span. The 2021 Physics noble prize went to Syukuro Manabe and Klaus Hasselmann, "for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming". They believe in the interaction between fastmoving weather and the slow-ing-evolving climate. The other half of the noble prize went to Giorgio Parisi "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales".

Slow/Fast Stochastic Systems. Suppose a quantity of interest interacts with many others, evolves at a much faster pace, and exhibits ergodic properties without necessarily diminishing in magnitude. In this case, we have a two-time-scale slow/fast system. Let ε denote the time separation scale. For example, consider a human compared to a dog's lifetime, $\varepsilon = \frac{1}{7}$. For the galaxy and human comparison, $\varepsilon \sim 0$. On the time scale of the slowly evolving object, the fast ones are not tractable, but they could leave a visible, persistent influence through ergodic averaging.

A multi-scale system of stochastic differential equations can offer mathematical explanations and predictions for the evolution of multi-scale systems.

$$\dot{x}_t^\varepsilon = F(\varepsilon, x_t^\varepsilon, y_{\frac{t}{\varepsilon}})\dot{\xi}_t + G(\varepsilon, , x_t^\varepsilon, y_{\frac{t}{\varepsilon}}) + H(\varepsilon, x_t^\varepsilon, y_{\frac{t}{\varepsilon}}).$$

Here y_t is a stochastic process, in the equation $y_{t/\varepsilon}$ is the fast variable, while x_t^ε is the slow variable.
Over 1 unit of time, the fast variable would have covered almost an eternity. The goal is to quantify its impact on the slow variable and obtain an autonomous approximate equation—the effective motion. Seeking the effective dynamics of the slow variable as the separation of their time scales becomes large is the fundamental question in this problem.

Perturbation to conservation laws. In mathematics, slow/fast systems arise from small perturbations to a dynamical system with conserved quantities. For example, consider a Hamiltonian function $H : \mathbb{R}^2 \to \mathbb{R}$,
and a small perturbation of the Hamiltonian system:

$$\dot{x} = \frac{\partial H}{\partial y}, \qquad \dot{y}(t) = -\frac{\partial H}{\partial y} + \varepsilon \dot{W}(t)$$

As $\varepsilon \to 0$, the solutions naturally converge. With a change of variable, in the action-angle coordinates, we get $I = \frac{H}{\omega}$, $I_t = -\varepsilon y_t \circ dW_t$. Changing time

$t \to\mapsto t/\varepsilon$, quivalent to considering I_t over the large time interval $[0, \frac{1}{}epsilon]$, we have

$$d\tilde{I}_t^\varepsilon = -\frac{1}{\sqrt{\varepsilon}}\tilde{f}(\tilde{I}_t^\varepsilon, \tilde{\theta}_t^\varepsilon) \circ d\tilde{W}_t$$

where $\circ$ denote Stratonovich integration, which unlike Itô integral, obeys the chain rule. Here $\tilde{W}_t$ is another Brownian motion, and we have written $y_t = f(I_t, \theta_t)$. This is clearly represents a system with a slow variable $\tilde{I}_T^\varepsilon$ and a fast variable θ_t^ε). As $\varepsilon \to 0$, one finds an effective motion describing the evolution of the Hamiltonian.

A second example of a slow /fast system is given by the evolution of a particle with small mass *m*,

$$\dot{x}_t = \frac{1}{m} y_t,\ dy_t = -x_t + g(x_t) + dW(t).$$

As the mass $m \to 0$, Ornstein-Uhlenbeck found that x_t^ε converges to a Brownian motion (the Kramers-Smoluchowski limit).

A third example arises from perturbations to a completely integrable stochastic Hamiltonian system, on a symplectic manifolds of dimension 2*n*,with *n*-Poisson commuting Poisson Hamiltonian vector fields H_i. One may consider a perturbation cross their invariant level sets:

$$dz_t^\varepsilon = \sum_{i=1}^n X_{H_i}(z_t^\varepsilon) \circ dW_t^i + \varepsilon V(z_t^\varepsilon)dt$$

where W_t^i are independent Brownian motions and X_{H_i} the symplectic vector fields associated with H_i. For $\mathbb{R}^{2n}$ with the trivial symplectic structure, $X_{H_i} = J\nabla H_i$ where $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ is the canonical skew symmetric matrix.

A fourth example is given by the perturbation of geodesics on the orthonormal frame bundle of a Riemannian manifold *M* of dimension *n*:

$$du_t^\varepsilon = H_{e_0}(u_t^\varepsilon) + \sum_{i=1}^n A_i(u_t^\varepsilon) \circ dW_t^i,\ u_0^\varepsilon = u_0$$

where H_{e_0} is the horizontal vector field such that the solution of $\dot{u}_t^\varepsilon = H_0(u_t)$ with the initial frame $u_0 : \mathbb{R}^n \to T_{\pi(u_0)}M$ projects to a geodesic with initial speed $u_0(e_0)$ and initial position $\pi(u_0)$. The fields A_i are vertical obtained by translating a family of skew symmetric matrices generating $\mathfrak{so}(n)$. Indeed, we can take an orthonormal basis of $\mathfrak{so}(n)$. After separating the slow and fast variables, we found the slow variable leads to, in the $\varepsilon \to 0$ limit, leads to a time changed Brownian motion on the manifold and their horizontal lifts.

Further interesting examples can be set on the Hopf fibration and indeed on homogeneous manifolds.

Stochastic Averaging and Homogenization for Markovian systems.
The theory of Stochastic Averaging and Diffusion Creation theory, within the framework of Markovian processes, has a long history. The first can be described as a dynamic theory of Law of large numbers,

$$\int_0^t f(y_{\frac{s}{\varepsilon}})ds \to t \int f d\mu$$

the second corresponds to a functional Central Limit Theorem:

$$\frac{1}{\sqrt{\varepsilon}} \int_0^t f(y_{\frac{s}{\varepsilon}}) \to \Sigma W_t \qquad (0.1)$$

for a function *f* that is centered with respect to the stationary measure of the Markov process y_t. The 'center' condition is used to solve the Poisson equation $\mathcal{L}h = f$, thereby isolating the highly oscillating terms.

Both fields have been active areas of research since the 1950's. The latter, also been known as (time) homogenization, has been very popular since Kipnis-Varadhan's work (1986) on this topic and the subsequent applications to hydrodynamics problems with simple exclusion processes (Kipnis, Olla, Varadhan 1989).

Functional Limit theorems and homogenization for non-Markovian with LRD. By contrast, there have been few landmarks in the study of slow/fast non-Markovian systems. The non-central limit theorems are well known to statisticians and some probabilists at the level of convergence in distributions. For example, consider the stationary 1-dimensional fractional Ornstein-Uhlenbeck processsolving the equation

$$dy_t^\varepsilon = -\frac{1}{\varepsilon} y_t^\varepsilon\, dt + \frac{\sigma}{\varepsilon^H} dB_t^H. \qquad (0.2)$$

$H \in (0,1) \setminus \{1/2\}$, The case $H = \frac{1}{2}$ corresponds to the Ornstein-Uhlenbeck process. If $H < \frac{1}{2}$ the processes is highly oscillatory, and the super-diffusivity needs to be tamed. For $H > \frac{1}{2}$, assuming that *f* is centred is no longer sufficient to guarantee a functional limit theorem of any kind. The trick is then to take *f* to be rougher, meaning that they should belong to the tail chaos expansions or certain order *m*, where $H^*(m) = m(H-1)+1 < \frac{1}{2}$, to have a chance for (0.1) to hold.

For functions with a lower Hermit rank *m*, it is still possible to obtain a scaling limit, with $\frac{1}{\sqrt{\varepsilon}}$ replaced by a number $\alpha(\varepsilon, H, m)$ depending on *H* and *m*. The limits would be Hermit processes.

We found that, to tackle the homogenization theory, it is fundamental to obtain a strong functional limit theorem in the rough path topology, by enhancing the processes to rough paths. We term this as rough functional limit theorems. From here one, one can use techniques from the theory of Rough Paths. The effective theory is much more colourful compared to the Markovian theory.

Convergence to equilibrium. The non-stationary theory for solutions of SDEs driven by a fractional Brownian motion differs naturally from the Markovian system, as stationary in the sense of having the same distribution at all times do not translates into shift invariance on the path spaces, and it is not possible to prepare the initial condition for the solution to be invariant as the system looking into its history. The rate of convergence cannot be easily obtained by taking solutions with different initial conditions.

Fractional averaging, and homogenisation for slow variables driven by fBM. Consider the simple yet illustrating equation for the slow variable

$$dx_t = f(x_t, y_{\frac{t}{\varepsilon}})dB_t^H$$

on $\mathbb{R}$, where *f* and the fast variable *y* are as good as we wish. In particular, consider y_t being a stationary Markovian process with a spectral gap.

On the stochastic averaging front, with $B_t^{\frac{1}{2}} = W_t$, x_t^ε converges, in law, to the Markov process whose *diffusive part* is $\Sigma \frac{\partial^2}{\partial x^2}$ where $\Sigma = \int f^2(x,y)\mu(dy)$.

The $B_t^H = dt$ case corresponds to *H* = 1, then solutions of

$$dx_t = \varepsilon^{-\frac{1}{2}} f(x_t, y_{\frac{t}{\varepsilon}})dt$$

converge weakly to a Markov process with diffusioncoefficient given by $\Sigma = \int (f\mathcal{L}^{-1}f)(x,y)\mu(dy)$.

By contrast, if $H > \frac{1}{2}$, we found that solutions of

$$dx_t = \varepsilon^{\frac{1}{2}-H} f(x_t, y_{\frac{t}{\varepsilon}})dB_t^H$$

converge, in probability, to that of $dx_t = \bar{f}(x_t)dB_t^H$ where $\bar{f} = \int f(\cdot, y)\mu(dy)$.

For $H < \frac{1}{2}$, slowing down the super-diffusion; the solutions of

$$dx_t = \varepsilon^{\frac{1}{2}-H} f(x_t, y_{\frac{t}{\varepsilon}})dB_t^H$$

converges to a Markov process whose diffusivity is given by

$$\Sigma := \frac{1}{2}\Gamma(2H+1) \int (f\mathcal{L}^{1-2H} f)(x,y)\,\mu(dy).$$

The same holds for $H > \frac{1}{2}$, assuming $\int f(x,y)\mu(dy) = 0$. This latter case corresponds to homogenization. Note that the formulas of the two classical theories are united in this above formula.

In conclusion, the problem with long range dependent noise has only recently been considered rigorously and forcefully in stochastic analysis. However, there are already some promising developments.

Written by: Xue-Mei Li (EPFL)

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Article on https://arxiv.org/a/li_x_9.html

A conversation with Marcos Mariño

Marcos Mariño is a Professor of Mathematical Physics at the University of Geneva. He has a joint appointment in the Department of Mathematics and in the Department of Theoretical Physics.

He has held postdoctoral positions at Yale, Rutgers and Harvard, and he was a junior staff researcher at the CERN Theory Division.

His work focuses on the mathematical aspects of quantum field theory and string theory, and in particular on their interfaces with modern geometry and topology.

To be a mathematician or a physicist, how does one decide?
Well, being a mathematical physicist, you can say that I have not decided, because I do both. When I was young it was a difficult thing for me to decide between math and physics, until I realised that mathematical physics was a way of not having to decide. There are different types of mathematical physicists, some are closer to mathematics and some closer to physics. The type of mathematical physics I do is closer to physics. And indeed, I have a degree in physics, I do not have a degree in math. The mathematical aspects of my work are very important, but I consider myself more of a physicist.

When and how did you get interested in mathematical physics?
It happened very early in my life, and it was all due to outreach. My family comes from an artistic and literary background and there is no one else in my family doing science. But I was very influenced by a TV program by Carl Sagan called Cosmos. It was a series of documentaries about science in general and I was greatly impressed by a chapter where Carl Sagan explained special relativity. I was 11 or 12 years old at the time, and this completely converted me to science. I knew then that I wanted to be a physicist.

How did you choose the academic path?
I grew up in an intellectual environment where going into academia was a natural choice. For me, it was clear from the very beginning that I will study and do my PhD and continue on this path. If I wanted to be a scientist, that was the way to achieve it. So, once I decided to do physics and mathematics, the academic path was obvious.

Quanta magazine published an article about your work titled, "*How to Tame the Endless Infinities Hiding in the Heart of Particle Physics*".

What are you working on right now and what impact could your work have on the future?
One topic I am interested in is to understand some of the more formal aspects of quantum field theory. This is related to the topic of the Quanta article and the ERC Synergy Grant I got in 2018, together with Jorgen Andersen, Bertrand Eynard, and Maxim Kontsevich. In order to do calculations, one of the main tools we use in physics is perturbation theory. We know however that perturbation theory is not the final answer, and perturbative series need to be complemented with something else. The nature of this something else is a very interesting problem, since we do not know very precisely what it is. So, what I have been working on these past few years is to understand from a more mathematical point of view how to go beyond perturbative series in physics and also in mathematics, because perturbative series appear in mathematics as well.

I believe these are very fundamental questions. Quantum theory can be formulated in terms of the path integral invented by Feynman, which is supposed to give a mathematical description of quantum field theory, quantum mechanics, and string theory. But we don't really know what this object is. There are many mathematicians who have tried to describe it rigorously, but I have been using this different perspective based on perturbative series, which is sometimes called the theory of resurgence. One nice aspect of this perspective is that you find many interesting intermediary results. That is one of the important aspects of science: you usually have a long-term goal, but it is very important to get interesting partial results along the way, even if you don't reach your goal. And thankfully, we have been finding many of these.

My work is in a sense very experimental. You take an idea from physics and use mathematics as your laboratory.

In general, my approach to mathematical physics is different from other people within SwissMAP. The starting point in traditional mathematical physics is a physical theory that uses mathematics, but in a way which is not really rigorous. Then mathematical physicists use mathematical tools to make this physical theory rigorous. My approach is rather the opposite: I start from ideas and techniques in physics and try to see whether they can lead to new insights in mathematics. This was a very successful approach pioneered by Edward Witten, the famous physicist and fields medallist.

So one of my goals is to better understand the mathematical structure of quantum field theory and string theory, but I am also using ideas from these two areas of physics to better understand problems in geometry, topology, and algebra. This has led to surprising results that are not easy to obtain otherwise. I don't produce many theorems, I produce mainly conjectures, but conjectures are very important for the progress of science since they articulate research programs. They are also very useful to mathematicians because they can transform them into theorems. And quantum physics has turned out to be an incredible machine that produces many interesting conjectures.

What have been the most rewarding or favourite moments in your career so far?
Fortunately, there have been many rewarding moments in my career. When you do research, there are always great moments where things fall into place and you make a discovery, or something finally works. It is difficult to choose a specific one, but a good example is a problem I worked on when I was a postdoc of Greg Moore in the United States. Our first finding was that, according to quantum field theory, four-dimensional spaces have to satisfy some special properties or constraints. This was very surprising and no mathematician had suspected this, because this idea was coming directly from physics. After discovering this idea, we decided we wanted to test it. I spent some time collecting all the ways in which mathematicians have constructed four-dimensional spaces, and I tried to see if I was able to violate these constraints found with physics. This proved to be impossible: every day I would use a new technique from mathematics to construct four-dimensional spaces and try to see if I could evade these constraints. And every time I tried to evade them, I failed, and found that the constraints from physics hold true. This was a very rewarding result because it felt as though you found a new physical law for four-dimensional spaces. We then reformulated this law as a mathematical conjecture that was eventually proved by geometers who are experts in the field of four-dimensional topology.

So my work is in a sense very experimental. You take an idea from physics and use mathematics as your laboratory. You don't prove theorems, rather you make experiments with mathematical objects. And it is very satisfying when you find that your experiments work.

What have been the greatest challenges you had to face?
Of course, being a researcher is a permanent challenge. But I believe an important challenge in my life was related to my background. I did my bachelor and PhD in Galicia, Spain, which is where I grew up, but this is not the epicentre of the world in terms of research. And this puts one at a disadvantage. It is not the same thing to get your PhD in Princeton than to get it at a second or third-rate university. In the latter case, you receive a worse education and have a worse professional network, because you are less connected to the important people in your field. It is a difficult challenge to overcome the limitations that come from your background, in order to make it to the top places where you can do good research.

So this was a structural challenge that I had to address. And I think that people do not talk enough about this. Where you grew up, and where you have done your studies or PhD, are important factors that have an enormous impact in your life. There was an article in Nature in 2022 that showed that 20% of the universities in the world produce professors for 80% of the universities. Most university professors have been educated in a very select group of universities, so if you have received your education in a peripheral university, it is simply more difficult to get to the top. I am surprised that people dealing with diversity and inequality in academia don't even talk about this, which is a massive source of inequality in the initial conditions of academic careers.

Life is not predetermined and I have essentially overcome this challenge, but it has had nevertheless some side effects for me. For example, there are some topics in which I received a very bad education, as the courses I took were not very good. Of course, you can study on your own afterwards, but it is important to get a good education from good teachers at the right moment in your life. The fact that I didn't get a very good education has been a handicap.

You have been involved in outreach activities. Can you share some examples, including challenges faced and how they were addressed?

What do you find most rewarding about outreach?
I have to say that my view of outreach has changed over the years, and my current view might be interesting for those who are involved in outreach activities. In the typical outreach activity, scientists give a talk about their research in a simplified way, mostly about current cutting-edge results. This is of course a very good thing to do, but it also has shortcomings. In a nutshell, what happens is that many of the people who go to outreach conferences about cutting-edge research – like black holes, string theory or cosmology – do not really know how science is done. Some of them believe in bogus pseudo-sciences and in all kinds of conspiracy theories. After a popular talk I gave on quantum geometry recently, a woman in the audience asked me if we can use quantum physics to explain homeopathy. We know that homeopathy is ruled out by elementary physics and chemistry, so this example shows that there is an unbalance between the sophistication of some of our outreach and the public, who in many cases is not sufficiently educated about science at the more basic level.

So I believe there should be more outreach focused on basic issues in science, such as how to reach a scientific conclusion, what are cognitive biases, why some things cannot be trusted since they violate the basic principles of science, why pseudo-sciences which are popular do not really make sense, and so on. This type of scientific education would be very important in order to develop critical thinking in our society.

Concerning traditional outreach, I think that transmitting and making an impact on others is the most rewarding aspect. As I explained before, my scientific career is the result of outreach and popular science. There were no scientists in my family, so if it weren't for people doing outreach, maybe I would have become a philosopher or a philologist. But thanks to outreach, I became a mathematical physicist.

That is what is so rewarding about outreach – you can change people's lives. And even when you don't change their lives, you expose them to great ideas and their lives get richer. This is by the way also true for scientists themselves, since for me one of the best things about being a scientist is to be in a permanent and intimate dialogue with the scientific tradition.

It is remarkable how people from different universities, and different regions, have come together through SwissMAP. It has created a sense of unity and a place for great communication and exchange of scientific questions and strategies.

What advice would you give to a PhD student who wants to pursue an academic path?
Academia has changed a lot since I started my studies. There is less faith in the search for truth and in the scientific method, and even within academia there are some tendencies that try to undermine the traditional way of doing science, which for me is still the right way to go. And this worries me a lot. So the advice I would give to a PhD student, or a beginning researcher, would be a traditional one: you have to work very hard. The PhD period is a moment in life where you are in front of a set of problems that you are supposed to solve, and it is crucial to solve them. Some of these problems might be hard, so this might be a very frustrating period as well, and people should be well aware of that. Failure is one of the possible outcomes, and it is not a good idea to try to find excuses when you don't solve a problem, you have to be prepared in case this happens. At least in mathematics and theoretical physics this is a period in your life where you are mostly on your own. Of course, you have your supervisors, but essentially you have to face this challenge by yourself.

I have the feeling that people are finding more and more excuses for not being able to address these challenges, and this is worrisome. External factors play a role, as I was pointing out before, but one cannot blame everything on them. At some point, it's just you and your problem. In an interview in 2021, the French physicist Édouard Brézin put this in a very clear way, and he says that once you have the material conditions to devote your entire time to solving a problem, the limitations are essentially your own. It is also going to be a difficult task, and very different from working a normal 9 to 5 job. It is something that will take over your life. And there is no way to sugarcoat it, it will be tough. Sometimes it will be very exciting and you're going to be incredibly happy when you solve a problem. And sometimes you're going to be in despair and feel sad and depressed. But that's all part of the experience. Research cannot be transformed into a "safe space" where nothing bad happens. It is more like a roller coaster, with ups and downs.

What impact do you think SwissMAP has had on the scientific community?
SwissMAP has created a coherent and diverse mathematical physics community in Switzerland that would probably not exist in the same way otherwise. It is remarkable how all the people from different universities, and different regions, have come together through SwissMAP. It has created a sense of unity and a place for great communication and exchange of scientific questions and strategies. This is really impressive. I also believe that this community has made an impact on the rest of the world because the world has been impressed by this community which is very diverse but at the same time able to work together.

There is also the beautiful legacy that SwissMAP will leave behind in the form of the SwissMAP Research Station. It has become one of the most precious research stations in the world for mathematics and physics. I would like to take this opportunity to congratulate the people within SwissMAP who have worked hard to make this possible. They did an amazing job.

It is well known that you love movies and know a lot about them. You were also involved with the Ciné-club Universitaire.

What is it exactly about cinematography that you like so much?
I love cinema, but I also love literature and I am interested in many other things. This might not be very good for a researcher, as it distracts you from your work, but you only have one life and there are so many interesting things to discover. Cinema and literature have always been very important to me. In a sense, this is the legacy of my upbringing, as I grew up in a very literary and artistic family. My father also worked at the ciné-club of my hometown when he was a university student.

George Eliot, an English writer from the 19th century and an extraordinary woman, gave a very good answer to what is interesting about literature, cinema and the arts. She wrote that "Art is the nearest thing to life; it is a mode of amplifying experience beyond the bounds of our personal lot". This ability to enlarge your horizons and your experience is to me the most fascinating thing about literature and cinema. Note however that artistic form is crucial in this endeavour. In that sense, art is not very different from physics to me, since physical theories talk about experience, but I only really admire them when they are formulated in a precise and deep mathematical form.

Working at the Ciné-club Universitaire was a great thing, since it allowed me to meet film directors and critics and to organise events. Although I'm no longer involved in the Cine-Club, I am planning to organise a course on cinema and science in the University of Geneva for the academic year 2025-2026. I will try to put these two subjects together and see what happens. The idea is to teach cinema to scientists, and at the same time, to teach some science to students in the humanities. It will be a double challenge because you have to address both audiences.

In regards to literature, what is your favourite genre and what are you currently reading?
I don't have a specific favourite genre but I mostly read classics. Currently, I'm very interested in 19th century English and American literature, specially George Eliot, Henry James and Edith Wharton, who are among my favourite writers. I don't read a lot of contemporary literature. Being a scientist, however, I have been interested in science fiction, especially Philip K. Dick, Stanislaw Lem and the Strugatsky brothers. I also try to avoid reading in translation and focus on books written in languages that I can read (Spanish, Portuguese, French and English), although I have read a lot of German and Russian literature in translation for example. At this moment I have just finished "Wise Blood" by Flannery O'Connor, who was a female writer from the United States. This is a short novel that inspired a John Huston movie of the same title.

Conversation with Marcos Mariño
Geneva/Online

Interviewed by Maria Kondratieva
On behalf of NCCR SwissMAP

Olga Trapeznikova

Presenting my research

My research focuses on the study of the topology of moduli spaces of semistable vector bundles on a Riemann surface, which play an important role in a wide range of subjects: from enumerative geometry to non-abelian gauge theories in physics. The Verlinde formula is an expression for a key topological invariant of these moduli spaces. I will briefly explain the parabolic variant of this formula in the simplest, rank-3 case.

We fix a compact Riemann surface Σ of genus *g* and denote by Δ the triangle in $\mathbb{R}^3$ shown in Figure 1. For a generic $c \in \Delta$ there exists a smooth compact moduli space $P_0(c)$ of rank-3 degree-0 parabolic bundles over Σ, and there is a natural way to associate to an integer *k* > *0* and a vector $\lambda \in \mathbb{Z}^3$ satisfying $\lambda_1 + \lambda_2 + \lambda_3 = 0$ a line bundle $\mathcal{L}(k;\lambda)$ on $P_0(c)$. The *parabolic Verlinde formula* is the following expression for the Euler characteristic of the line bundle $\mathcal{L}(k;\lambda)$: for $\lambda/k \in \Delta$ generic

(1) $$\chi(P_0(\lambda/k), \mathcal{L}(k;\lambda)) = 3^g (k+3)^{2g-2} \cdot \sum \frac{i \cdot \exp(2\pi i\,((\lambda_1+1)x_1 + \lambda_2 x_2))}{\prod_{i<j} (2 \sin \pi (x_i - x_j))^{2g-1}},$$

where the sum is taken over the finite set of points $x = (x_1, x_2, x_3 = 0)$ satisfying $0 < x_i - x_{i+1} < 1$, $(k+3)x \in \mathbb{Z}^3$ and $x_i - x_j \notin \mathbb{Z}$.

The Verlinde formula has attracted a lot of attention over the years, and has a number of different proofs. In collaboration with Andras Szenes, I gave a new proof of this formula, which uses only the basics of geometric invariant theory, and stands out with its technical simplicity. Our proof is based on the following idea: first, we show that both sides of the equation (1) are piecewise polynomial functions in variables *k* and λ, then using geometric properties of the moduli spaces $P_0(c)$ (for the left-hand side) and a residue calculus (for the right-hand side) we deduce that both functions satisfy certain properties which determine them uniquely, and thus we obtain the equality (1).

I used a similar geometric approach to give a complete description of the enumerative K-theoretical invariants of these moduli spaces and presented explicit formulas for the Euler characteristic of tautological vector bundles on the moduli spaces $P_0(c)$.

Parabolic bundles also appear in my ongoing project on the calculation of another topological invariant of moduli spaces of vector bundles: their Betti numbers. Denote by $M_d(r)$ the moduli space of rank-r degree-d semistable vector bundles on a compact Riemann surface Σ. When r and d are coprime, $M_d(r)$ is smooth, and the Betti numbers of this moduli space were calculated by Harder and Narasimhan, Atiyah and Bott. In the case of arbitrary rank and degree, $M_d(r)$ is singular, and the relevant problem is the calculation of its *intersection* Betti numbers. The study of this invariant is a classical problem, which goes back to the works of Frances Kirwan in the 80's.

In joint work with Camilla Felisetti and Andras Szenes, we give a complete description of these structures via a detailed analysis of the Decomposition Theorem applied to a map from a smooth moduli space of parabolic bundles $P_0(c)$ (with a special choice of parameter c) to $M_0(r)$. We also obtain a new formula for the intersection Betti numbers of the moduli spaces $M_0(r)$, which has a clear geometric meaning.

These two rather different projects have shown that a technical notion invented in the 70's, parabolic bundles, can help to solve a number of central problems in modern enumerative geometry.

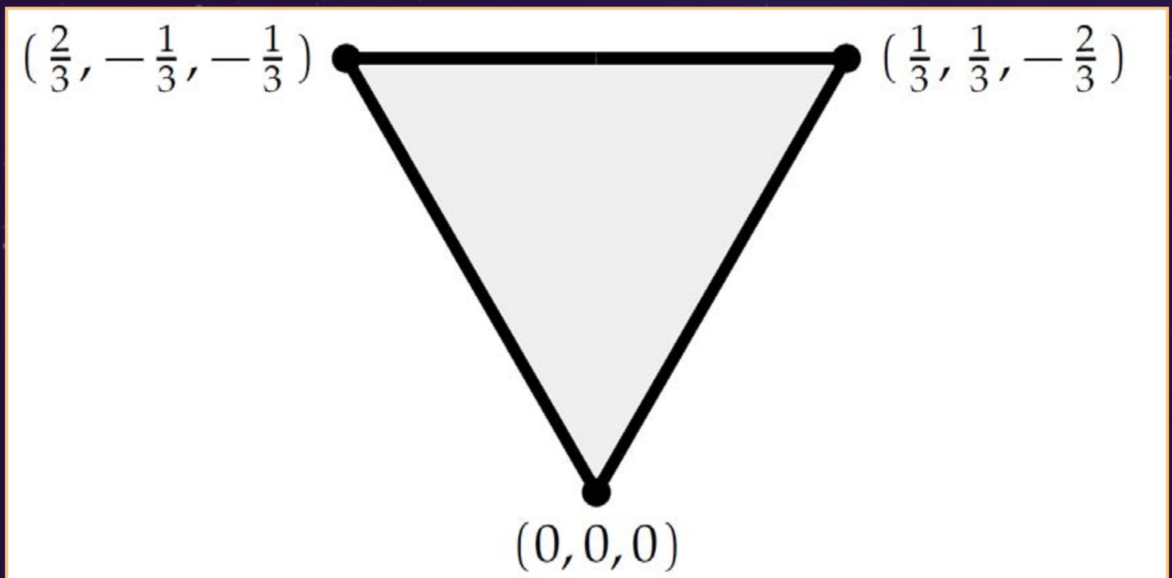

Figure 1. Credit: Olga Trapeznikova

Author: Olga Trapeznikova
UNIGE, A. Szenes's Group

Adriàn Sànchez Garrido
Towards a holographic description of wormholes

Black holes (see Figure 1) are very strange beasts. They describe a region where the fabric of spacetime is so stretched that its curvature diverges in its center, which is surrounded by an event horizon that splits causally the interior from everything outside: Not even light can escape. In addition, they may pinch spacetime and open up wormholes, shortcuts between spacetime regions that would otherwise be far apart (or even disconnected).

If gravity is combined with the laws of quantum mechanics, black holes are even weirder. For instance, they radiate and can eventually evaporate, spitting back the information that they swallowed during their lifetime in the remnant radiation. Giving a microscopic description of this and other phenomena, such as the evolution in time of the wormhole behind the black hole horizon, is a challenge that can be tackled with the help of the AdS/CFT correspondence (also called the holographic principle), according to which generic gravitational theories in Anti-de Sitter space are dual to quantum field theories with conformal symmetry but no gravity, in one less spatial dimension, which can be thought of as being defined on the boundary of spacetime.

The wormhole joining two black holes in different universes might be understood, in the framework of the holographic duality, as a tool that entangles the quantum states on the respective boundaries. This proposal is, however, not quite right: Entanglement entropy in black hole scenarios ceases to increase at time scales at which the wormhole throat still has a large scope of growth. Nowadays, the most widely accepted proposal for the holographic dual of the wormhole length is, rather, that of complexity, which is a quantum-information theoretic notion that aims to describe how complicated a quantum state is, i.e. how difficult it is to prepare.

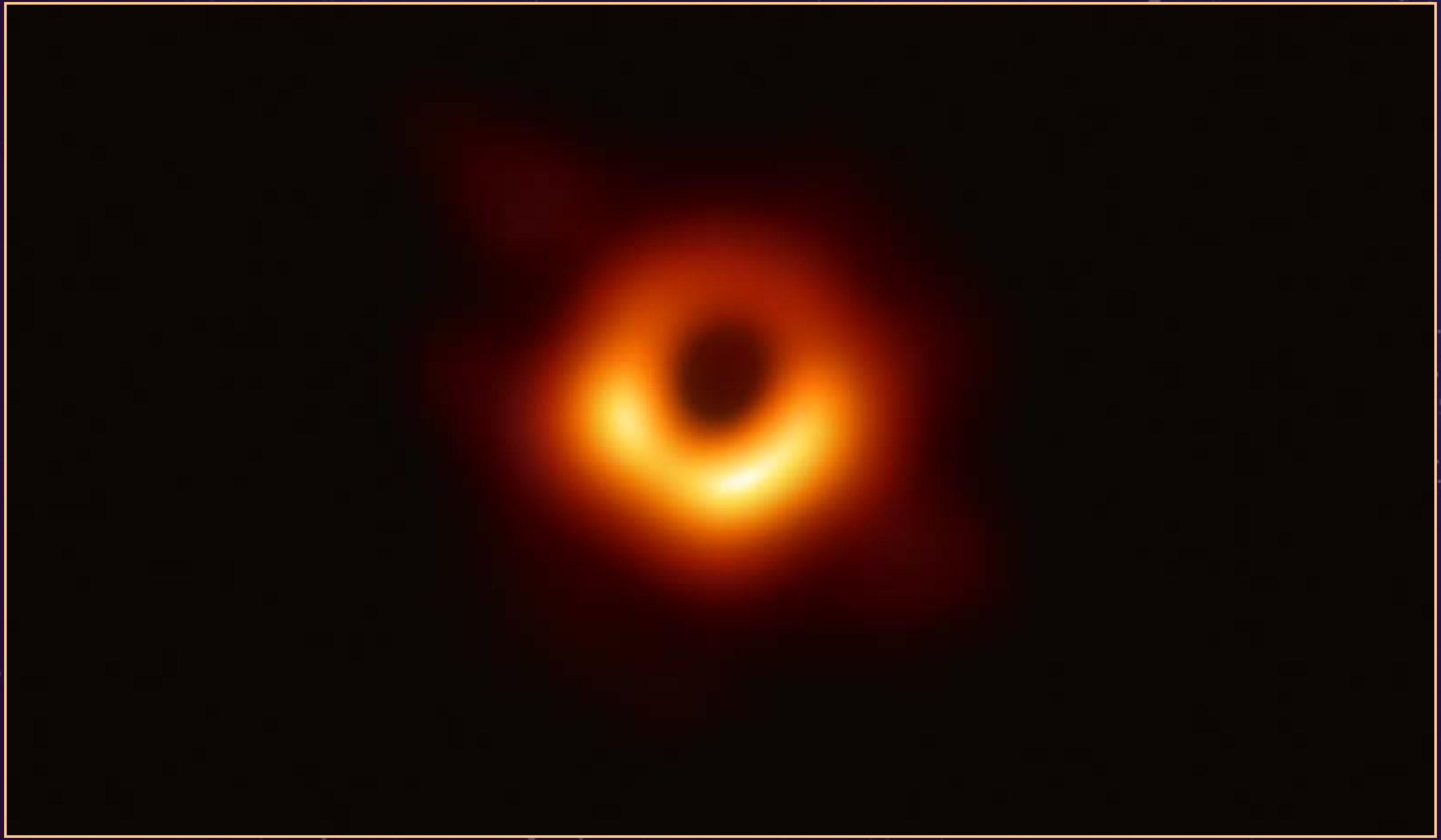

Figure 1: Black hole image captured by the Event Horizon Telescope. Nowadays there is certainty that black holes do exist out there, elevating the questions on their nature and properties from theoretical chimeras to actual unsolved mysteries of nature.
Credit: https://eventhorizontelescope.org/press-release-april-10-2019-astronomers-capture-first-image-black-hole

There are many ways to define consistently a notion of quantum complexity. The most immediate one, adapted to quantum circuits, consists on fixing a reference state together with a set of quantum gates, in order to quantify the complexity of a certain target state as the minimal number of gates required to reconstruct it out of the reference state, up to some tolerance. However, this proposal is not suitable for use in holography, as the ad-hoc external choices needed to define it can't be matched to intrinsic properties of the gravitational theory.

During my PhD I have investigated, together with Ruth Shir, Eliezer Rabinovici and my supervisor Julian Sonner, a newly proposed measure of quantum complexity, dubbed Krylov complexity, which unambiguously quantifies the complexity of a state as its average position on a basis of the available Hilbert space which is by construction progressively explored during time evolution. This Krylov basis (also dubbed Krylov chain) is built by suitably orthogonalizing nested powers of the time-evolution generator acting on the initial condition by the means of the Lanczos algorithm, a technique that was already well-known in the framework of Krylov methods for the diagonalization of linear operators, pioneered by the Russian mathematician and engineer Aleksey N. Krylov in the 1930s.

In a series of papers [1–3] we performed heavy numerical simulations in order to compute the Krylov complexity of observables up to times exponentially large in system size in chaotic quantum systems such as the Sachdev-Ye-Kitaev (SYK) model, and in strongly-interacting integrable ones, like the XXZ spin chain. We were able to access the complexity saturation regime and showed that Krylov complexity is sensitive to chaotic dynamics, growing as fast as possible and saturating at higher values for chaotic models, while in integrable ones complexity growth is hindered by a novel localization phenomenon on the Krylov chain. These properties are crucial for an eventual holographic application, since black holes are known to be chaotic systems.

Complementing this, in [4] we established, for the first time, an analytical correspondence between Krylov complexity of a particular state (the infinite-temperature thermofield double state) and bulk wormhole length in a concrete instance of low-dimensional holography, the one relating the SYK model and two-dimensional Jackiw-Teitelboim gravity. This is a very exciting result which represents a step forward in the construction of the complexity item in the holographic dictionary. Further research is still required, but it seems that Krylov's techniques for the diagonalization of matrices have turned out to be the right language to describe the quantum properties of wormholes!

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Dmitry Krachun
Why do we study lattice models?

In statistical physics one often models some real life macroscopic phenomena in terms of microscopic interactions of particles using simplifying assumptions with respect to the interaction, lattice structure and randomness. The most prominent example is the Ising model, which was originally introduced to model the behaviour of a magnet, and in which one assumes that a magnet is made of a huge collection of tiny magnets occupying vertices of a part of a cubic lattice, each having spin either +1 or -1. The spins are sampled randomly according to Boltzmann distribution, simple interactions between neighbouring magnets then govern the behaviour of the whole system. Another similar example is that of Bernoulli percolation. Introduced to model how water spreads through porous medium, such as a sponge, this model is audaciously simplistic: One assumes that a sponge is made of tiny tubes connecting neighbouring vertices of the cubic lattice and each tube is open (i.e.water can go through it) independently with probability $p \in (0, 1)$.

One way to appreciate the complexity of the model is to look at samples of configurations, see Figure 1 for some samples of the Bernoulli percolation on the plane. When looking at the regions of a (2-dimensional) sponge making caveats for the water, i.e. connected by tiny open tubes, we observe that the picture is drastically different depending on whether the parameter p, which is the density of open tubes, is below 0.5, equal to 0.5, or above 0.5. This is, in fact, an illustration of the *phase transition* phenomenon familiar to everyone who has had a chance to see how water boils and becomes vapour.

Another intriguing feature is the fractal structure of the clusters at the critical point $p = 0.5$. While with visible mesh size (such as one in Figure 1 on the left) the direction of the lattice are clearly visible, once the mesh size becomes increasingly smaller the structure of the clusters becomes independent from the directions of the lattice. In fact, this phenomenon is one the reasons making the lattice models relevant in the first place, for the structure of a real sponge is clearly isotropic. Mathematically, we could phrase this phenomenon by stating that the model is *rotationally invariant*, meaning that rotation of the lattice preserves the behaviour of the model in the small mesh-size limit.

In physics, this phenomenon is explained by the so-called *universality hypothesis*, which roughly speaking states that in the vicinity of a critical point, the behaviour of the model is essentially independent of its microscopic details. The universality hypothesis also suggests that a given oversimplified model, such as the Ising model, belongs to a large class of systems, including realistic ones, which all share the same qualitative and even quantitative features. Hence, by studying the simplest of the representative of this universality class one, in fact, gets an insight into the behaviour of the real systems. Unfortunately, mathematical understanding of the universality hypothesis is lacking and it has been proved only in the simplest of cases, for instance, the case of a random walk, where Donsker's invariance principle tells us that the random walk converges, in the small mesh-size limit, to the Brownian motion.

Large part of my PhD thesis is devoted to the proof of the rotational invariance of the critical planar FK models, which is a class of models generalising both Bernoulli percolation and the Ising model mentioned at the beginning. In fact, the main part of the proof consists in showing universality of the model among a certain class of planar graphs, each possessing a flip symmetry with respect to a different line. Combining all these symmetries one then gets the rotational symmetry of the model.

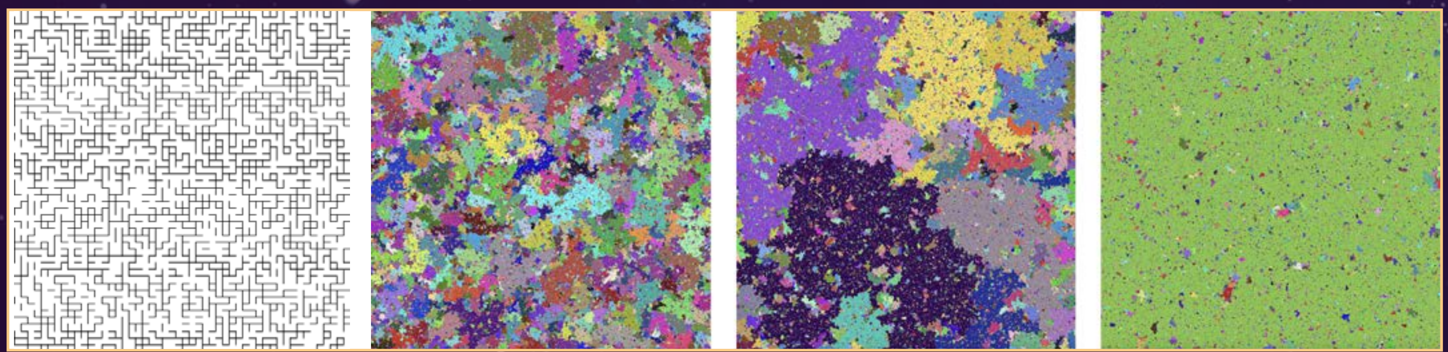

Figure 1: On the left: a sample of a percolation configuration on a 50 by 50 square with parameter p = 0.51, black edges correspond to open tubes; on the right: three samples of the percolation configuration on a 600 by 600 square with clusters coloured in various colours. The parameter p is subcritical for the first picture (p = 0.49), critical for the second picture (p = 0.5) and supercritical for the last picture (p = 0.51); first image taken from wikipedia, second image taken from Raphaël Cerf's website.

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Cone Types in Cayley Graphs: What, How, Why?

The notion of *cone types* in a finitely generated group was first introduced by Cannon in the 80s and was used in particular to show his celebrated result of the rationality of growth functions for all Gromov hyperbolic groups [3]. Since then, they have been widely studied and have led to some interesting results for various classes of groups and graphs, as we will discuss.

1. What are cone types?
To fix the notation, let $G = \langle S \rangle$ be a group generated by a finite symmetric set *S*.
We like to represent it by a graph *Cay(G, S)*, called the *Cayley graph*, whose vertices are the elements of *G* and $g, h \in G$ are joined by an edge whenever $g^{-1}h \in S$. The fundamental idea of geometric group theory due to Gromov is to look at a group as a metric space endowed with the *word metric*, defined by $d_s(g, h) = l_s(g^{-1}h)$, where $g, h \in G$ and

$$l_s(g) = \min\{n \in \mathbb{N} \mid \exists s_1, \ldots, s_n \in S \text{ s.t. } g = s_1 \ldots s_n\}.$$

It corresponds to the length of any shortest path in *Cay(G, S)* connecting the elements *g* and *h*.

The aim is to describe the asymptotic geometry of this graph by looking at its "cones"; more precisely, the *(word) cone type* in a Cayley graph is defined for $g \in G$ as

$$T(g) = \{h \in G \mid l_S(gh) = l_S(g) + l_S(h)\}.$$

A particularly nice situation is when the Cayley graph has *finitely many* cone types. Gromov hyperbolic groups are the only class of groups known to have finitely many cone types with respect to *any* generating set [3, 4]. Another well known class of examples that satisfy this finiteness property is *Coxeter groups*, with respect to Coxeter generators; this was proven by Brink and Howlett [2]. In this case, we encode the cone types as the states of a directed graph (in fact, a finite state *automaton*), which provides a combinatorial tool to work with the Cayley graph. Furthermore, the cone type automaton being finite-state is equivalent to the language of reduced words of *G* in *S* being *regular*.

2. How to determine them?
In our work, we focus on a particular family of Coxeter groups, namely *hyperbolic triangle groups*. Such a group is given by a presentation of the following form

$$\Delta(l, m, n) = \langle L, M, N \mid L^2 = M^2 = N^2 = (LM)^n = (MN)^l = (NL)^m = e \rangle$$

and yields a tessellation of the hyperbolic plane, obtained by successively reflecting a triangular tile of angles $\frac{\pi}{l}$, $\frac{\pi}{m}$ and $\frac{\pi}{n}$ in its sides. The

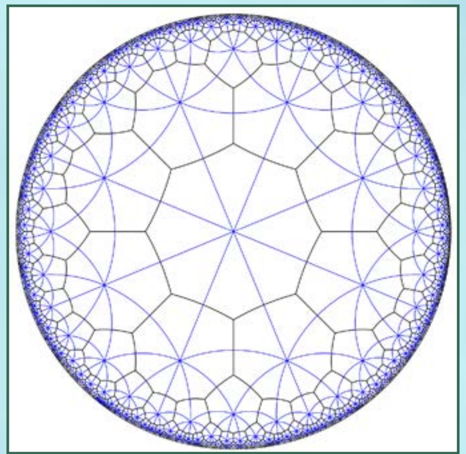

Figure 1. The dual tessellations associated to Δ(4, 4, 4): its triangular tessellation in blue and its Cayley graph in black.

dual tessellation, composed of *2l*–, *2m*– and *2n*–gons, corresponds to the Cayley graph associated to this presentation. It will be convenient to consider the Poincaré unit disc as our geometric model for the hyperbolic space; see Figure 1.

Let us present a simple formula due to Parkinson and Yau [9] that yields the cone type of any given $g \in G$. It relies on the property of these groups that all generators are involutions and correspond to reflections in the hyperbolic plane.
Indeed, given $g \in G$ and $s \in S$ then we have

(1) $T(gs) = s\{w \in T(g) \mid l(sw) = l(w) - 1\}$,

which we interpret geometrically as the reflection by *s* of the set of all elements of the previous cone type that are shortened by left multiplication by *s*. We apply (1) inductively to each element of the *n*–sphere of *Cay(G, S)* for increasing values of *n* until no new cone types appear, since the cone type of a vertex determines its neighbours on the *(n+1)*–sphere (i.e., its *successors*) and their cone types. As an example, applying this algorithm to Δ(4, 4, 4) leads to 21 cone types, which can further be organized in 6 types depicted in Figure 2.

3. Why are they of interest?
The study of a group's cone types is interesting as it offers a new insight into its geometry and has links to many topics of geometric group theory. Indeed, a finite set of cone types is a valuable combinatorial tool: they can be used to study certain asymptotic characteristics of infinite groups or graphs using only this finite set of data. For instance, they can be used

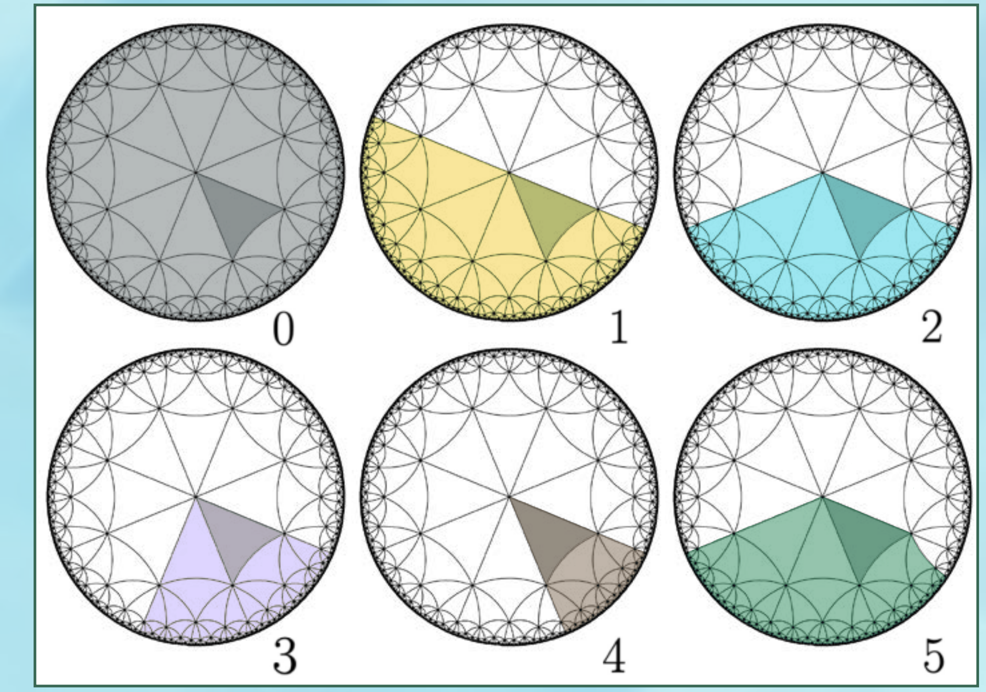

Figure 2. The 6 equivalence classes of cone types of Δ(4, 4, 4).

to compute the associated *growth series* [3, 1] and are at the centre of algorithms producing tight bounds on the *spectral radius* by both above and below [8, 5, 6]. The latter is interesting as it is a characterization of amenability, by the celebrated theorem of Kesten. An additional motivation comes from an experimental approach to a conjecture of Kaimanovich and Le Prince about the harmonic measure associated to the random walk on a Fuschian group, proposed recently by Pollicott and Vytnova [10].

Finally, we are currently interested in extending the application of the cone types structure to study other asymptotic characteristics of hyperbolic triangle groups, but also of other classes of groups and hyperbolic tessellations [7].

Written by Megan Howarth
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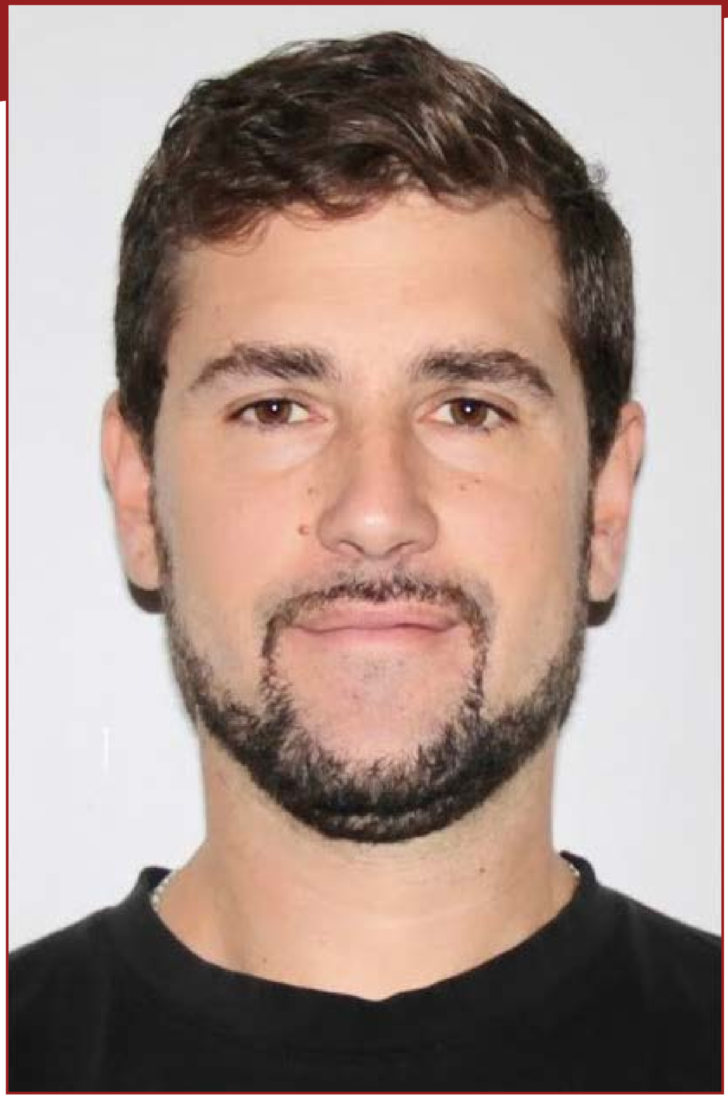
Alumni Corner

Alexandre BELIN

(University of Milano Bicocca)



Alexandre Belin received his PhD from McGill University. He completed postdocs at Stanford University (ITP), Amsterdam University, as well as a postdoc at CERN and UNIGE in J. Sonner's group.

He is currently an assistant professor at the University of Milano Bicocca in the department of physics Giuseppe Occhialini. He has started his own research group and continues his work on quantum gravity, combining tools from quantum chaos and the conformal bootstrap program.

SwissMAP continously strives to maintain a strong connection between all past and present members. Our alumni corner presents inspiring stories from some of our previous members.

What's your favourite part about being a physicist or physics in general?

The freedom. Of course, as you get older, you get less freedom in your scientific career because of various constraints, but I would still say freedom. You can always think about the things that you want to think about. You're always free to explore new things and you have no boundaries. It is you who decides what you find interesting, and what you want to do, and this is quite amazing.

And my other favourite part is, that you're never doing the same thing over, and over again. You keep discovering new things you didn't know about and new things to learn. In this sense, it is a very free and non-repetitive job.

Can you briefly explain your research?

Broadly defined, my research focuses on quantum gravity. Quantum gravity is the theory that is supposed to put together general relativity, which is our theory for understanding classical gravity and gravitational waves in our universe, together with quantum mechanics, which is how we think about fundamental interactions at the microscopic scale. Quantum gravity is the theory that is supposed to put those two things together. And as we've known for many decades now, it's a very difficult problem because they don't want to be put together. That is broadly speaking, what I work on.

Then more specifically, I work on holography or gauge/gravity duality. Which is a very powerful tool to understand quantum gravity because it says that a theory of quantum gravity is dual or can be alternatively described by an ordinary quantum theory without gravity that lives on its boundaries. So, that's what I work on.

If we zoom in yet again within holographic duality, for the past few years, I have been trying to understand how to describe quantum properties of black holes. Bekenstein and Hawking told us that black holes are thermodynamic objects, that have a lot of entropy. And so, the question is, what is the microscopic origin of this entropy. And how do we understand what the microstates of a black hole are, what properties do they have, and how can we use insights from quantum chaos to think about that question.

How did you come across this topic of research and why did you choose it?

Since the beginning of my PhD, I've been working on holography and quantum gravity. That was always the general theme. What I have been doing more precisely however, has shifted throughout the years. Indeed, this connection with the field of quantum chaos is something new and was not talked about when I started my PhD. It has only become a bigger topic seven or eight years ago.

And how did I get into it? Well, I attended a few talks by people who were working in this field and I thought it was very exciting. As I was a postdoc in Stanford, Professor Stephen Shenker, who is an amazing physicist, taught a class on the topic. This was my first contact with the field of quantum chaos. I didn't start working on it straight away though, but it was always in the back of my mind. I understood how useful it could be to think about black holes, which was a popular subject at the time. I gradually started deepening my knowledge and expertise on the subject, and finally, started working in this field seriously, three or four years ago.

The field is a very dynamical thing, with interests that spike at different moments in time. But for me, it was a more gradual process. I started by first getting more familiar with it and as I realised how powerful it was, that was when I started seriously working on it.

How exactly will your research impact the scientific world?

We are all always trying to do our part, as much as we can. I hope to make progress within quantum gravity of course. Quantum gravity is a very complicated question, that many people have been working on for many years. One always hopes that the breakthrough is around the corner, and we will be able to fully solve it. That is something we hope for, but of course, we don't know what will happen in the future.

There is however a part of the future that we can expect, and this is one of the reasons why I find physics so exciting. There are continuously new connections that happen between subfields of physics. Things that you were not studying before, because you thought were part of a separate field, become suddenly very connected to a problem you're working on. That's how I think about quantum chaos. One of the impacts that I hope my research can have, is to create new connections between fields that we thought were separate before.

One of the reasons why I find physics so exciting is, there are continuously new connections that happen between subfields of physics. Things that you were not studying before, because you thought were part of a separate field, become suddenly very connected to a problem you're working on.

And I hope to create these new insights going both ways. Either between the standard quantum chaos community, which is more condensed matter, and the quantum gravity and black holes community.

So, I hope to be able to answer some very hard questions. And we have answered some questions, to some extent. I also hope to make connections between different fields, that were not known before.

What did you mostly enjoy about SwissMAP?
My involvement in SwissMAP was very short because I was officially a SwissMAP member only for a couple of months. But before that, I spent 3 years at CERN and one at EPFL after. So, even though I was not officially a part of SwissMAP, I was still very involved in the activities. For example, we organised one of the first workshops for the SwissMAP Research Station back in 2021 during Covid. There were only 15 participants back then.

What I really enjoy about SwissMAP is that it works as a community. Switzerland is not a big country, even though the community has grown a lot, but SwissMAP has really put the community in contact in a way that it was not before. So, I think it is very successful on that level. And it was very fun to be a part of those activities. We all know each other now and we all discuss physics together. Either through events in Les Diablerets, or various other meetings that SwissMAP organised or helped to organise. This sense of community is what I enjoyed most about being a part of SwissMAP.

Who inspires you in the world of physics?
Many people inspire me. Another great thing about the world of physics is the way you can get inspired by other people, and then in turn inspire others yourself. Many of the people I have worked with, and indeed people I have not worked with, inspire me a lot. For example, the physicist I mentioned earlier in relation to quantum chaos, Steve Shenker. He is a fabulous physicist who gives very inspiring talks. He is one of the people I have learned from a lot. Another great physicist is Juan Maldacena. I have interacted with him throughout the years, and he always has amazing insights that inspire me to think about things in a deeper way. But I would also mention all my collaborators. The people I have worked with over the years, always find ways to inspire me and teach me new things.

Did you ever consider the industry at any point?
Yes, I did, because I didn't have a job for a long time. I received another faculty job offer a long time ago that I had to turn down for personal reasons, but then for many years, I had no job security as many others in academia. Of course, you can close your eyes on it and hope everything works out. But most people don't have that ability and think about their future on a day-to-day basis, and I am one of those people. So, I also considered the industry during that time. I like teaching a lot and always thought that my road out of academia would probably not be industry, but rather teaching at maybe a high school level. But I am very happy that things worked out in academia.

The industry, and specifically machine learning, is attracting physicists by the second. And not just people who are choosing the industry because they have no future in academia. There are many people who have very successful paths but decide to go into industry before they get a faculty job. But there are also many people who have faculty jobs and decide to leave to do machine learning. Currently, I don't think I will be one of them, but who knows what the future will hold. In a sense, I was lucky to get a job in academia. So, the only reason I would leave and go to industry would be by choice. If I end up doing it, then it would be because I want it, as nothing is forcing me to go. But as things are right now, I'm very happy doing research.

What advice would you give to a PhD student pursuing an academic career?
Don't do it! (laughs)

No more seriously, to someone who wants to do a PhD, I would say yes, absolutely do it. I think a PhD is a great thing to do regardless of your future choices. It is a way to mature intellectually. You become a better thinker in the broad sense of the word. And it's very good for the world in general, to have a lot of people who did PhD's and had to think a lot about very hard problems.

Academia is then the next step if you decide to make a career there. Of course, I'm joking when I say don't do it, but I'm also only half-joking. The problem is that the world has evolved a lot, but academia hasn't changed, and this has created an unbalance. I meant what I said when I described what I loved about being a physicist, but there are now many other jobs that have these advantages as well. But they also have more job security without having to worry about what the next postdoc will be. They are also very fun, and the pay is usually better. I believe people should consider this more now. The academic job market was always rough, but it is a lot rougher now, than say, 20 years ago. And the alternatives are not the same as they were back then. This is why we see many people leave academia even though they had a good place there. I hope that academia will also evolve in a positive way. Then I can tell everybody, "Yes, do it, it's great and there's nothing to worry about".

What I really enjoy about SwissMAP is that it works as a community.

Do you have any hobbies?
I have tons of hobbies. I was a DJ for 10-15 years. Of course, my job and family don't allow for that kind of lifestyle anymore, but I still love music. But I'm also a very outdoors person. I hike a lot in the summer and snowboard in the winter whenever I can find the time. I like ski touring and mountaineering. So, music and outdoor activities are where I spend the free time I find outside of family and work.

Conversation with A. Belin
Formerly: UNIGE, J. Sonner's Group

Interviewed by Maria Kondratieva
On behalf of NCCR SwissMAP

Nuriya NURGALIEVA

(University of Zürich)

Nuriya Nurgalieva (formerly ETH Zurich, R. Renner's Group) completed her Bachelor's degree in Physics at Moscow Institute of Physics and Technology, and her Master's degree at ETH Zürich. She did her PhD in the research group for Quantum Information Theory at ETH Zurich.

Nuriya's research interests are in the areas of quantum information and quantum foundations. She is currently managing upcoming outreach projects for Quantum Year 2025 at the University of Zürich.

What's your favourite part about being a mathematician or maths in general?
Quoting Ursula K. Le Guin, "If a book were written all in numbers, it would be true. It would be just. Nothing said in words ever came out quite even. Things in words got twisted and ran together, instead of staying straight and fitting together. But underneath the words, at the center, like the center of the Square, it all came out even. Everything could change, yet nothing would be lost. If you saw the numbers you could see that, the balance, the pattern. You saw the foundations of the world."

Can you briefly explain your research?
I have worked in a variety of topics encompassing different areas of research such as quantum foundations, quantum information and quantum clocks. In quantum foundations, I have worked on the topic of multi-agent epistemic paradoxes where we looked at whether theories remain logically consistent if you choose to model simple reasoning systems (agents) within them along with taking into account their reasoning, and what are the conditions that are necessary for such consistency. In quantum clocks, we looked at clocks as quantum-classical systems where quantum machinery produces classical ticks, and investigated their different facets: from basic properties to how one can use them as a reference or for time estimation.

How did you come across this topic of research and why did you choose it?
I specialised in biophysics in my Bachelor studies, but when I arrived to ETH to do my Master degree, I decided to try out something new – so I signed up for the Quantum Information Theory course (taught by Renato Renner at the time). I was hooked right from the first lecture – the concept of finding and formally defining a theory from a set of few conceptual axioms, so popular in the field of quantum information, was novel for me and aligned well with how my mind works.

> Don't put yourself under unnecessary pressure. Take time for yourself, your hobbies, and your friends; your research is not what defines you.

What did you mostly enjoy about SwissMAP?
During my time at SwissMAP, I was a part of the SwissMAP Research Station committee, where we were tasked with evaluating workshop and schools proposals based on their inclusivity and scientific merit. This allowed me to get an idea of different research directions pursued at SwissMAP, and also gain experience in evaluating (and writing!) grants and proposals.

How exactly will your research impact the scientific world?
I hope it gives an idea of how physical theories (in particular quantum mechanics) work, how systems like agents (simple reasoning machines) and clocks can be described within them, and what are the consequences of having such objects in the theory.

Did you ever consider the industry at any point?
My main area of interest lies in science communication and outreach, which is what I do in my current position. But if this career path does not work out, I would definitely consider switching to industry!

What advice would you give to a PhD student pursuing an academic career?
A general advice: don't put yourself under unnecessary pressure. Take time for yourself, your hobbies, and your friends; your research is not what defines you.

Do you have any hobbies?
In my spare time, I like doing sports such as running, cycling and rock climbing. My current focus is on long distances – right now I am training for my first marathon. I am also an avid fiction reader and dabble in illustration and design (scientific and otherwise).

Conversation with N. Nurgalieva
Formerly: ETHZ, R. Renner's Group

Interviewed by Mayra Lirot
NCCR SwissMAP

Upcoming Events

SRS SwissMAP Research Station in Les Diablerets

Scientific Program 2025

Workshops & Conferences

WORKSHOP ON RESURGENCE, WALL-CROSSING AND GEOMETRY
January 12-17
J. Andersen (Southern U. of Denmark),
B. Eynard (U. Paris-Saclay), M. Kontsevich (IHES),
M. Mariño (UNIGE).

ALGEBRA AND QUANTUM GEOMETRY OF BPS QUIVERS
January 19-24
W. Li (Institute of Theoretical Physics),
P. Longhi (Uppsala U.), M. Mariño (UNIGE),
B. Pioline (Sorbonne U. Paris).

NEW FRONTIERS IN EXTREMAL AND PROBABILISTIC COMBINATORICS
January 26-31
A. Shapira (Tel Aviv U.), B. Sudakov (ETH Zurich).

STATISTICAL MECHANICS, ALGEBRA, AND GEOMETRY
February 2-7
S. Shatashvili (Trinity College), J. Sonner (UNIGE),
E. Verlinde (U. Amsterdam).

STOCHASTIC EQUATIONS AND STOCHASTIC DYNAMICS
February 9-14
M. Gubinelli (Oxford U.), X-M. Li (EPF Lausanne).

WORKSHOP IN MATHEMATICAL PHYSICS
February 23-28
S. Smirnov (UNIGE).

EFFECTIVE THEORIES FOR MANY-BODY SYSTEMS OUT OF EQUILIBRIUM
May 11-16
L. Delacrétaz (U. Chicago), B. Doyon (King's College),
T. Sasamoto (Tokyo Institute of Technology),
J. Sonner (UNIGE).

MATHEMATICAL FOUNDATIONS OF BIOLOGICAL ORGANIZATION
May 18-23
J-P. Eckmann (UNIGE), K. Kruse (UNIGE).

HELVETIC ALGEBRAIC GEOMETRY SEMINAR (HAGS) 2025
June 1-6
R. Pandharipande (ETH Zurich), A. Szenes (UNIGE).

POLYLOGARITHMS, HOMOLOGY OF LINEAR GROUPS, AND STEINBERG MODULES
June 8-13
A. Kupers (U. Toronto), P. Patzt (U. Oklahoma),
D. Rudenko (U. Chicago), T. Willwacher (ETH Zurich).

TOPOLOGICAL SYMMETRIES AND DEFECTS IN QUANTUM FIELD THEORY (WORKSHOP & SCHOOL)
June 15-20 & June 22-27
J. Brown (U. Edinburgh), A. Cattaneo (UZH),
M. Del Zotto (Uppsala U.),
J. Plavnik (U. Indiana Blumington),
C. Teleman (UC Berkeley).

THE AMPLITUHEDRON: STRUCTURE, COMBINATORICS, AND POSITIVE GEOMETRY
June 29 - July 4
N. Arkani-Hamed (Institute for Advanced Study),
T. Lakrec (UZH),
L-K. Williams (Harvard U. & Radcliffe Institute).

INTERSECTIONS OF TOPOLOGICAL RECURSION, CONFORMAL FIELD THEORY, AND RANDOM GEOMETRY
August 24-29
N. Aghaei (UNIGE), C. Guillarmou (U. Paris-Saclay),
R. Kashaev (UNIGE), N. Orantin (UNIGE),
E. Peltola (Aalto U. & U. Bonn).

INTERNATIONAL WORKSHOP ON AUTOMORPHIC FORMS
August 31 - September 5
C. Burrin (UZH), L. Garcia (U. College London),
Y. Li (TU Darmstadt), R. Zuffetti (TU Darmstadt).

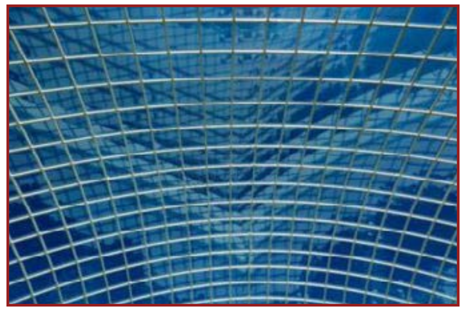
In 2025

Maths à PartaG
In this series of lectures, researchers and specialists in the field of mathematics share their passion with the general public. From the rich history of mathematics to the cutting-edge research conducted at the University of Geneva's Section of Mathematics, and to the pursuit of recreational mathematics in education, there is something for everyone! Participation is free and open to all. No prior knowledge is necessary; your curiosity is the only prerequisite. http://unige.ch/math/GEM

Past Events

Video recordings of the following previous events are available through playlists on our NCCR SwissMAP YouTube channel.

2024



Conformal field theory 3 ways: integrable, probabilistic, and supersymmetric
(Jan 21 – 26)
The purpose of this workshop was to bring together young researchers and experts from three communities, to share their techniques and insights, and work towards a unified framework.

Other 2024 recordings available:
- Les Marmottes 2024 - Filles et Maths (Apr 8 – 12)
- Quantum Topology Biennial (QTB): focus on representation theory (Jan 14 – 19)

2023

- Quantisation of moduli spaces from different perspectives (Sept 24 – 29)
- Categorical Symmetries in Quantum Field Theory (Conference and School) (Aug 27 – Sept 8)
- Effective theories in classical and quantum particle systems (June 18 – 23)
- Finite dimensional integrability in mathematical physics (June 11 – 16)
- Analytic techniques in Dynamics and Geometry (May 28 – June 2)
- Geometric and analytic aspects of the Quantum Hall effect (May 7 – 12)
- Integrability in Condensed Matter Physics and Quantum Field Theory (Feb 3 – 12)

Awards & Grants

Hugo Duminil-Copin

French Presidential Council of Science and the Académie des Sciences de Paris

Hugo Duminil-Copin is one of the twelve distinguished scientists on the panel, that will make up the French Presidential Council of Science. And on the 12th of December 2023, Hugo Duminil-Copin, was appointed to the Académie des Sciences de Paris.

Alba Grassi

FNS 2023 Starting Grant

We are pleased to announce that Alba Grassi was awarded a Starting Grant from the SNSF for the project entitled *Quantum Curves and Black Holes.*

Elise Raphael

EMS Mathematics Outreach & Engagement Committee

Our Science Officer Elise Raphael joined the European Mathematical Society (EMS) Mathematics Outreach & Engagement Committee.

Vincent Tassion

2023 Golden Owl

Congratulations to our member Vincent Tassion (ETH Zurich) for receiving the 2023 Golden Owl. The Golden Owl honours lecturers which have provided exceptional teaching and motivates them to continue with their excellent teaching.

SwissMAP Junior Researcher Prize

SwissMAP and G-Research invite nominations and applications for the first SwissMAP Junior Researcher Prize

Deadline for nominations & applications: September 30, 2024.

* Up to three prizes will be awarded once a year to PhD students or Postdoctoral researchers for important scientific achievements in Mathematics and Theoretical Physics, supported by G-Research.
* The call is open to all PhD and postdoctoral researchers (maximum 5 years after PhD completion) in mathematics and theoretical physics enrolled in a Swiss institution at the time of application.
* We welcome self-nominations as well as nominations by PhD advisors and postdoctoral mentors.

New Members

Claire Burrin

UZH

We welcome our new SwissMAP member Claire Burrin (UZH). She is joining the SwissMAP Phase III Direction *Statistical mechanics and random structures.*

Her research interests are number theory, harmonic analysis, automorphic forms, dynamical systems and group actions.

Shota Komatsu

CERN

Welcome to our new SwissMAP member Shota Komatsu (CERN). He joined the *Holography and bulk-boundary correspondence* & *From Field Theory to Geometry and Topology* SwissMAP Phase III Directions.

Shota 's main research interest is string theory.

Francesco Riva

UNIGE

We welcome our new SwissMAP member Francesco Riva (UNIGE). He is joining the SwissMAP Phase III Direction *From Field Theory to Geometry and Topology.*

His research interests include: Quantum Field Theory, Effective Field Theory, Positivity (Unitarity/Causality) bounds, Particle Physics Phenomenology and Hamiltonian Truncation.

Puzzle Corner

1. Sport teams without twins

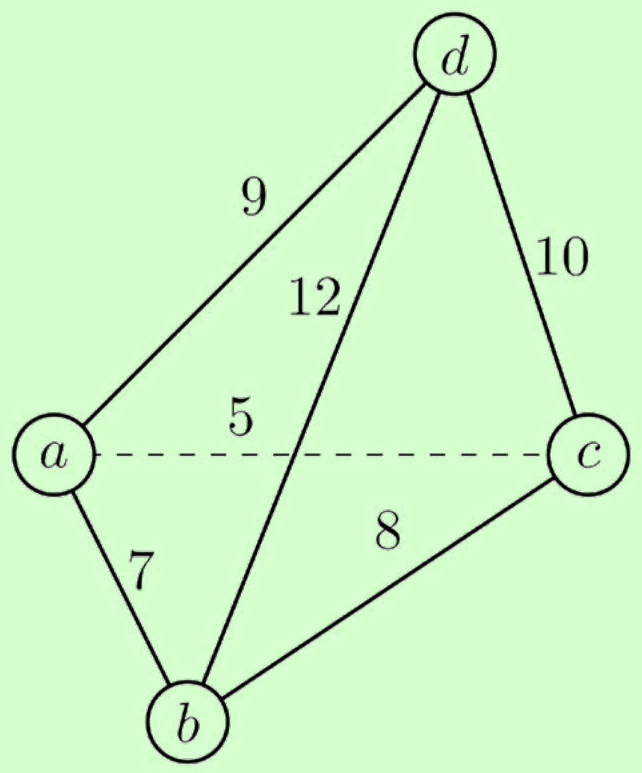
A sports teacher has a group of 6 pairs of twins. He wants to form teams so that no twins play together in any team.

a) In how many ways can he divide them into TWO teams of 6?
b) In how many ways can he divide them into THREE teams of 4?

2. Secret numbers of the pyramids

Secret numbers are written in the corners of a three-sided pyramid. The sum of the two secret numbers in the corresponding corners is written on each edge.

Determine the secret numbers of the pyramid in the picture below.



3. Dividability by 6

What is the largest number of natural numbers from 1 to 100 that can be selected so that the sum of any two selected numbers is dividable by 6?

4. The robot's number jumble

A robot has rearranged the numbers 1, 2, 3, 4, ..., 98, 99, 100 "numerically alphabetically" so that all the numbers starting with 1 are first, then the numbers starting with 2 and so on.

The beginning of the new row looks like this: 1, 10, 100, 11, 12, ...

How many numbers are in the same place as in the original order?

5. Prime differences

Determine the largest natural number n with the property that for every prime number p with $2 < p < n$, the number $n - p$ is also a prime number.

Answers

1. Sport teams without twins

a) 32 ways.
The 1st person can be chosen in 12 ways, the 2nd person in 10 ways (not she and not the twin), the 3rd person in 8 ways etc.

12 x 10 x 8 x 6 x 4 x 2 / 6! = 26 x 6! / 6! = 26 = 32

Alternatively: 1 is taken from each pair =64, but here each team was counted 2 times, i.e. 64 ⍰ 2 = 32 species

b) 960 ways.
For team 1 as above:

12 x 10 x 8 x 6 / 4! = This leaves 2 pairs of twins and 4 "singles".

For team 2, you have to take one from each pair of twins and "fill them": 2 x 2 × 4 x 3 / 2! = 24.

Team 3 is then clearly defined.

The 3 teams are thus counted 3! = 6 times

Then there are a total of 240 x 24 x 1 / 6 = 960 ways to choose teams of 4.

2. Secret numbers of the pyramids

a = 2, b = 5, c = 3, d = 7

The sum of the two numbers on the opposite edges corresponds to the sum of all secret numbers S.

We add up all the numbers on the edges that start from ONE corner (e.g. A) and call the sum SA.

This sum contains the secret number a three times and every other secret number once.

I.e. a = 1/2(SA - S) = ½ ((5 + 7 + 9) - (9 + 8)).

All other secret numbers are determined in the same way.

3. Dividability by 6

A maximum of 17 numbers.

If you look at the remainders of the numbers when dividing by 6, you realise that these remainders must be either 0 or 3 for all numbers.

The first idea would be to take the numbers that are dividable by 6. There is a set with 16 numbers.

However, if you choose the numbers with the remainder 3, you can increase the number of numbers by one.

The optimal set with 17 numbers is {3,9,15,21, ... ,87,93,99}.

4. The robot's number jumble

11 numbers: 1 and 80 - 89.

Of the numbers beginning with a 1, only the 1 retains its position.

Among the numbers that are ≥ 20, 10 numbers that begin with the same digit retain their position.

This means that if we have understood how many of the numbers 20, 30, . . . , 90 keep their position, then the next 9 will also keep their position.

If we check these numbers, we realise that the number 80 is in the right place (i.e. also all 81, 82, ..., 89).

This means that these 10 and the 1 remain, all other numbers "move".

5. Prime differences

n = 10.

If n>10, then one of the numbers n-3, n-5, n-7 will be dividable by 3.
n=10 works.

Puzzle contributed by:
Dmitrij Nikolenkov (ETH Zurich) | TagesAnzeiger's - Hier trainieren Sie Ihr Hirn | https://www.tagesanzeiger.ch/ (Folgen 343 - 347 des Zahlendrehers)

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Leading House

Co-leading House

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