SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

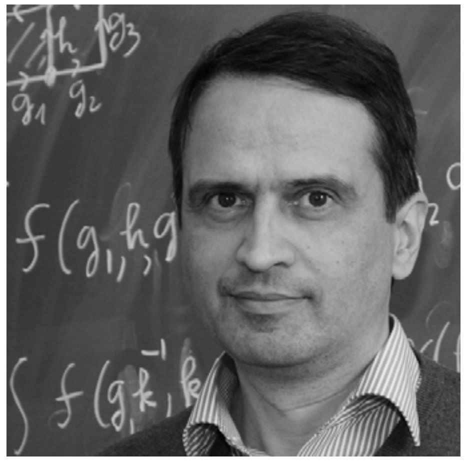
SwissMAP Perspectives

Issue 1 | Spring 2016

Contents

Directors' Note

Prof. Stanislav Smirnov
Director of NCCR SwissMAP



Prof. Giovanni Felder
Co-Director of NCCR SwissMAP

Dear Reader,

Welcome to this first edition of *SwissMAP Perspectives*.

SwissMAP is a Swiss National Centre for Competence in Research co-created by the University of Geneva and the ETH Zurich. It is an interdisciplinary research centre at the crossroads of mathematics and theoretical physics. In recent years, the interaction between these two fields has led to the creation of a new discipline where mathematical rigor and physical intuition merge in a natural way. Our goal is to bring our understanding of this field to a new level, which will have two-fold benefits: on one hand, it will help to make the description of nature mathematically more precise, and on the other it will lead to a deeper understanding of the mathematics in terms of which these physical ideas are described.
The research network of NCCR SwissMAP also includes the University of Zurich, EPFL, University of Bern, University of Fribourg, and CERN.

In summer 2016, SwissMAP will celebrate its two years of existence. This magazine illustrates the achievements and developments obtained from fall 2014 to spring 2016.
This issue highlights the conferences that were organised to celebrate the 100 years of Einstein's General Theory of Relativity. Both Zurich and Geneva hosted a series of conferences and our contributors were on-site to relate the events.
Starting in fall 2015, the first SwissMAP Master Class began. The program provides a small number of outstanding students Master-level classes in the field of planar statistical physics. Next year's Master Class is already in the making and will be focused on geometry, topology and physics.
A large number of events were organised by SwissMAP during its first year. During one of these, the Mirimanoff Lectures, Prof. Jürg Fröhlich agreed to sit down with us for a talk about mathematical physics.
This magazine also highlights future events of 2016, awards and grants received by some of our 40+ participants and a presentation of some of the new collaborators who have recently joined SwissMAP. Also, don't forget to test your math and logic skills in our puzzle corner.

We hope you enjoy this first issue of *SwissMAP Perspectives* and get a broader view of our research goals. If you would like to find out more about our achievements, please visit our website: http://nccr-swissmap.ch/

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Trajets Imprimerie & Copies Services

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100 years of General Relativity

If Albert Einstein could ride Mr Tompkins' relativity bike and slow down the flow of time up to last November, he would discover that his general theory of relativity, exactly 100 years after its first publication, doesn't look a day older. Multiple initiatives around the world have been occurring to celebrate this centenary, including in Switzerland.

In this context, a series of four conferences in Geneva "Les secrets de la gravitation – 100 ans de Relativité Générale", organised by SwissMAP in the evenings between the 24th and 27th of November 2015, opened the doors of this pillar of science to the broad audience. From the expert to the student, the philosopher to the simple curious; even some kids were present. The universal and interdisciplinary nature of this theory brings people together, regardless of their age or cultural background, because it stimulates our curiosity and touches upon those philosophical questions prone to every human since the dawn of time.

The conferences organised by the ETH in Zurich during the "Einstein Symposium" between the 12th and 14th of November 2015, were more technical. However, the audience was still varied and an overall view of the theory was also present.

Experts in the field met at the "28th Texas Symposium on Relativistic Astrophysics", which was held in Geneva between the 13th and 18th of December 2015. Following the morning plenary sessions and afternoon talks, a conference for a wider audience was also organised on the evening of December 15.

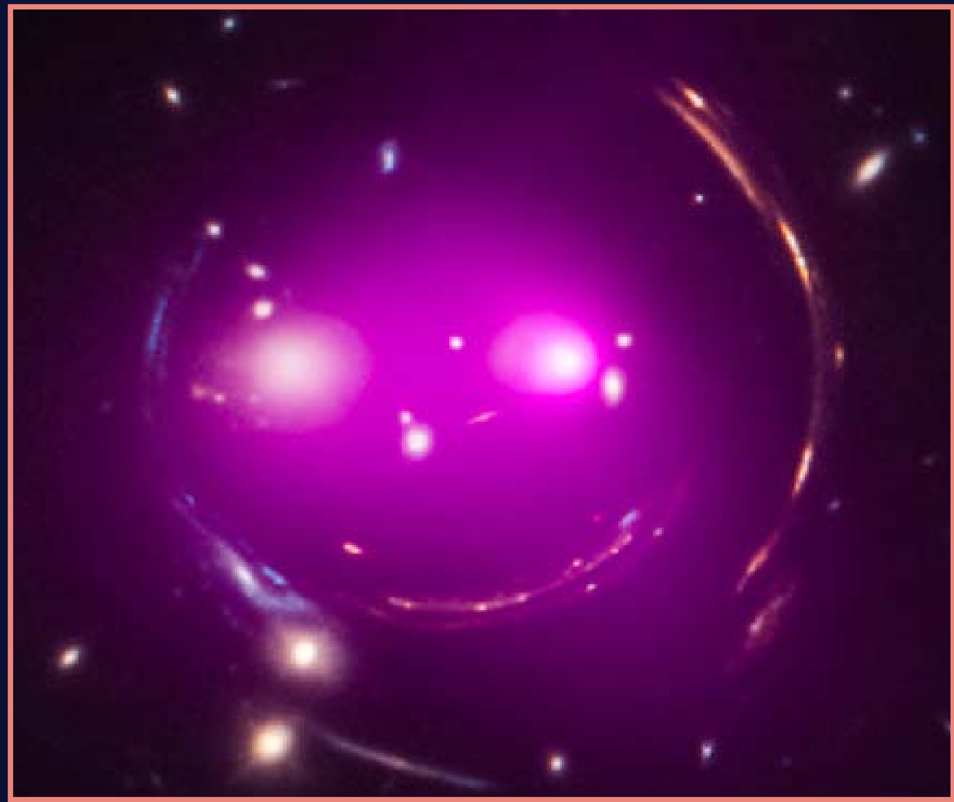
Although the theory of General Relativity (GR) was published in November 1915, its roots go back to at least one decade prior. Indeed, it was in 1905 that Einstein, with his limited version of the theory, broke the Newtonian concept of separated and fixed space and time: two observers moving with respect to one another have the same description of electromagnetic phenomena (this illustrates the covariance of the Maxwell Equations, which were only just finalised at the time). Indeed, this more general vision of space-time could naturally explain the constant value of the speed of light measured by Michelson and Morley at the end of the 19th century. The extended version of his theory arose from Einstein's wish to include the accelerating frames into his new vision of space-time, now considered as one entity. In particular, he wanted to take into account the presence of gravitation and managed to do so after many years of work, thanks to his motivation for depicting the surrounding world, which forced him to learn some new mathematics with the help of his friend and mathematician David Grossman.

Thus, the theory of GR was thought up and written within a handful of years by a man who was able to go beyond the thought pattern of his time, and answer a contemporary physics question (though seen as minor by many of his colleagues) in a simple and elegant way: how to make the physics we know invariant under any change of reference, even in the presence of gravity. However, it took several decades for the scientific community to grasp the range, implications and new questions that this theory entailed.

GR is not only a new tool for understanding gravity, but after a century, proved to have been – and continues to be – a multidisciplinary revolution of human thought. It is a good example of how mathematics and physics lean on each other to grow. If the latest advancements in mathematics at the time enabled a description of gravity in terms of the geometry of space-time, interpreting the solutions of this description gave rise to many developments, as well as predictions of some objects still abstract at the beginning of the 20th century, such as black holes and gravitational waves. Both of them were only indirectly detected last September, when the LIGO interferometers were able to detect a gravitational wave for the first time, right in the middle of the GR anniversary... Such a welcome gift!

GR is not only a new tool for understanding gravity, but after a century, proved to have been – and continues to be – a multidisciplinary revolution of human thought.

From an experimental point of view, much effort and progress has been done both in the technique and in the collaboration between experts from various fields. Whether it is to test the GR's predictions or to reach new and yet unknown limits. The universe surprises us each time, and with each new discovery, we become more and more conscious of our ignorance. And yet the vision of relativity remains valid, elegant, unbreakable, and continues to impress us with its simplicity, hidden behind a mathematical language so unfamiliar to the mere mortal. Einstein would probably be amazed at how much his predictions are still at the heart of research in fundamental physics.



Multiband view of the « Einstein's smiley » (2015): some arcs arising from the strong gravitational lensing effect, predicted by A. Einstein.
Credit: X-ray - NASA / CXC / J. Irwin et al. ; Optical - NASA/STScI

The Relativity revolution is visible across disciplines through four main themes of modern physics, all of which were presented during the various conferences: gravitational lensing, gravitational waves, black holes and cosmology.

In regard to gravitational lensing for example, scientists were sceptics at the beginning of the 20th century because it was not (yet) possible to observe the redshift due to the sun's gravity. And although Relativity could easily explain the advancement of the elliptical orbit's axis of the Solar System's planets, it was only decades later that one would notice the breath-taking precision with which this theory explains the deflection of light by the gravitation of celestial bodies such as galaxies. Einstein did not think one could observe this phenomenon other than the one created by the sun, because in his time the universe beyond the Milky Way was unknown and considered to be immobile. Today, gravitational lensing allows us to detect exoplanets and reveals that the main mass of the universe is made of matter that does not interact electromagnetically and hence is not made of atoms – "dark matter". It also allows us to reconstruct the dark matter's distribution in space, which shows that on a large scale, our universe has the structure of a mesh net. Without dark matter the universe would not be what it is, galaxies would never have formed, nor Earth nor life. Yet we have not yet been able to detect a particle of dark matter. Perhaps the LHC collisions in CERN will soon reveal this. Furthermore, the deviation (due to the curvature of space-time) of the trajectory of the oldest observable light, the cosmic microwave background (CMB), allows us to state that the universe is flat with a precision of 1/10000. If Einstein were present during this month of November, he would have noticed that neither the universe, nor human stupidity have yet reached their limits.

The universe surprises us each time, and with each new discovery we become more and more conscious of our ignorance.

Einstein was also doubtful of the existence of gravitational waves (GW). It took him a number of years and some arguments in 1936 with a prestigious American journal (anecdotal ever since), to realise that the mathematics behind his equations require a subtle interpretation and admit that these would lead to proving the existence of ripples in the fabric of space-time. But it was only in the 80's that the physicists Hulse and Taylor were able to prove the existence of GW for the first time. They measured how the orbit of a pair of pulsars decreases over time, showing that the energy loss can only be explained by taking into account the GW emission, exactly as it was predicted by the GR. The first historical detection of GW, however, was last year on September the 14th through the merger of two black holes (BH) of about 30 solar masses each. 2015 will be a highly remembered year. Not only for the

Photo taken during the conference "Les Secrets de la Garvitation" at UNIGE, Geneva
Credit: Maria Podpokaeva

first detection of a GW signal, but also because we were able to observe as close to the horizon of a black hole as ever before. More than a finish line or the fully deserved reward of fifty years of hard work of the scientific community, this detection represents the beginning of a new astrophysics era, testing gravity in its extreme conditions, i.e. where GR is more than just a correction to the Newtonian theory.

Before this detection, the existence of astrophysical black holes was mainly confirmed by observations in X-ray emissions from many active galactic nuclei (containing a supermassive black hole: ~$10^6 M_\odot$) or from some smaller stellar compact sources ($< 15 M_\odot$). The existence of some black holes having a mass of many tenths of solar masses remains unexplained today.

In addition to GW, scientists plan to observe the neighbourhood of black hole's horizons also using electromagnetic radio waves: by measuring the speed of the stars surrounding it, it has been proved that a supermassive black hole is present in the nucleus of the Milky Way. Astronomers will soon be able to "see" what happens near the horizon of this black hole using a worldwide network of radio detection pointed in the direction of the centre of the Milky Way: the Event Horizon Telescope (EHT).

Physicists love to challenge their theories with the hope to find their limits, and this is what they try to do with GR: any discord with the theory – never observed until 2015 – would be a precious clue to construct a new theoretical model. This new model would describe the gravity of high-energy – dominating at large scales – when it becomes comparable to other interactions, on a microscopic scale. We actually do not have a theory that generalizes Relativity and the Quantum Field Theory.

Thus, theorists seek for any observational evidence that could help them sort through the numerous models available. Indeed, a more general theory would be able to describe not only the physics beyond the horizons of black holes, but also the whole Universe at its very beginning: the other case where gravity matches microscopic physics.

Another context in which GR is of importance is in the study of the universe as a whole: modern cosmology. At the beginning of the 20th century, as the equations of the new-born GR were applied to the universe, the scientists realised that they do not apply to static solutions. At the time, the idea that the universe was either in expansion or contraction, but never static, was insurmountable even for such a revolutionary mind as Einstein's. In order to compensate for the effect of gravity on a static universe, he introduced a term in his equations – the Cosmological Constant. Yet, he was not satisfied with this addition as the resulting artificial "static" was unstable. Even more so when Edwin Hubble observed the universes expansion for the 1st time in 1924. Einstein qualified the "Cosmological Constant" as the biggest blunder of his life. Yet, once again, he was far from being able to predict that in 1998, observations of an acceleration of the universe's expansion by the "Supernova Cosmology Project" team would leed the scientists to reintroduce the Cosmological Constant. Though not to avoid the collapse of a static universe, but rather to prevent the slowing down of the universe's expansion, unavoidable if the universe is only made up of matter. Nowadays, the Cosmological Constant has become more noteworthy than ever as it models the acceleration of the universe. However, we still do not know the nature of the energy from which this acceleration originates. This is the reason we call it "dark energy". What are the physics of this mysterious energy that was weak at the beginning of the universe, but – as its density remains constant over time – has only recently become dominant (compared to the age of the universe)? Is this a new component of the universe, a sort of energy of the empty part of space? This would match the predictions of Quantum Mechanics, however the calculations for this density give a result that is 60 times greater in magnitude. How to consolidate this difference in values?

Another explanation for the universe's acceleration is that the graviton – the particle carrying the gravitational interaction – could have a mass, contrary to a photon. In this case, the scope of gravitation would be limited, and hence would not be dominant over very long distances. We will be able to test this when a GW detection will have an electromagnetic counterpart: a time delay between the electromagnetic signal and the GW signal of the same event. Showing that GW have a speed smaller than the speed of light would mean that the graviton has a mass.

Cosmology passed from being a mysterious and imprecise branch of Astrophysics to a science of high precision.

The celebrations of the GR century were a great occasion for a complete overview on the state of our knowledge and, most of all, the extent of our ignorance. It also helped us realise how within a few decades, cosmology passed from being a mysterious and imprecise branch of astrophysics to a science of high precision. Cosmology-related projects have multiplied, are being led by big collaborations between hundreds of scientists, and are often planned to last multiple decades. The analysis and interpretation of the results and the surprises that the universe has in store for us, are left to the next generations. Hence, it is vital that the younger generation is conscious of the implications of fundamental research. However, Relativity remains a relatively new theory and not very accessible, and modern cosmology is its' descendant. The general population, though quite curious, is usually little informed about these subjects and the public conferences for the GR centenary have played an important role in spreading the word.

In order to ensure the succession, SwissMAP aims to raise awareness in the younger generation by popularising these subjects in their education. For example, by introducing activities related to these subjects in the high-school program. This allows us to teach the fundamentals of mathematics to high-school students through motivating and interesting exercises, and to transfer, in a natural way, a sense of how the scientific approach has evolved during the last century.

Les secrets de la gravitation - GR100
24 - 27 November 2015, UNIGE
http://nccr-swissmap.ch/GR100

Einstein Symposium
12 - 14 November 2015, ETHZ
http://nccr-swissmap.ch/events/100-years-general-theory-relativity

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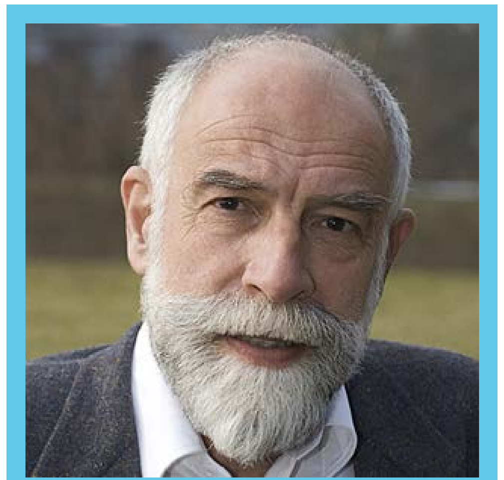
Conversation with Jürg Fröhlich



Jürg Martin Fröhlich is a Swiss mathematician and theoretical physicist. Since 1982 he has been a professor of theoretical physics at ETHZ, where he founded the Center for Theoretical Studies. He was awarded the Henri Poincaré Prize in 2009.
His research interests include: quantum field theory, precise mathematical treatment of models of statistical mechanics, theories of phase transition, the fractional quantum Hall effect, and non-commutative geometry.

During his visit to Geneva for the Mirimanoff lectures 2015, Jürg Fröhlich sat down with us for a conversation about mathematical physics, hiking and politics.

- You witnessed the development of mathematical physics through many years. How has the field evolved and what has changed since you started?

Mathematical physics is a field that develops mostly methods expected to be helpful to solve problems primarily arising in physics. A field like that is always in danger of decoupling. For example, it may decouple from physics, because there are plenty of wonderful mathematical questions people get enthusiastic about and then forget the physics that originally motivated those questions. Sometimes it also decouples from the mathematical main stream: it sort of takes off on a tangent that points towards somewhat sterile territory. And, depending on the period in history, mathematical physics has thrived and has been very successful; but then it has become a little dry and uninteresting. It has been oscillating forth and back ever since the field was created. I would say that, disregarding people of the 19th Century, such as Maxwell, Hamilton and Poincaré, mathematical physics has had its best period between 1905 and 1930, or so, because the great revolutions in theoretical physics at the beginning of the twentieth century came about by people who had a very intimate relationship with mathematics and, in this sense, could be called mathematical physicists. Then, after this fantastic period, mathematical physics went into hiding, and when theoretical physics emigrated towards the United States of America, which was during and after World War II, most of our famous colleagues there claimed not to like mathematics. At least, the official party line was that if one needs mathematics, one sits down and invents it, one doesn't have to study it. – So, what about the present? At present, it appears to me that mathematical physics is sent into exile in mathematics departments. Nowadays, it has become very rare that mathematical physicists are hired in physics departments – except for string theorists. But mathematical physicists of my brand do not appear to get jobs in physics departments anymore. This changes the field. I think, I always had a very close relationship to important problems of physics, and I talked to physics colleagues. People working in math departments are more likely to just study some mathematical aspects of a problem that, perhaps, originally came from physics. This is already visible in the way mathematical physics has developed during recent years. I think it's not the best period, but presumably, after a while, the pendulum will swing into the other direction again. So I'm not too worried about the future; I think there are plenty of good problems around and many interesting questions, and many of these questions are, I think, doable – one could surely make progress if one attacked them.

- Which direction of mathematical physics would you recommend to a student just starting his or her career?

I had many PhD students, and I had to give them advice about what kind of profile to develop. It is a known fact that mathematics departments are much bigger, because mathematicians have to give courses in essentially every department of a technical university. It is therefore easier to find jobs in mathematics departments than in physics departments. Thus, usually, if not almost always, I recommended to my students to attempt to develop a profile of a professional specialist in at least one mathematical discipline. Of course, I also always told them that, once they have a more or less stable job, and assuming they will still be interested in physics, they should try to have close contacts with physicists and find out what the physicists are interested in. I can tell you what my own interests are at the moment. I still have an interest in certain aspects of condensed matter physics. I used to work on the quantum Hall effect for rather many years, and I found the problems in this area extremely interesting, because this is an area where you could inject ideas from Conformal Field Theory, from Topological Field Theory, from abstract algebra, e.g., from the theory of tensor categories, and so on; and these ideas turned out to have really useful applications. Then there are those topological insulators that have fascinated me since

the '90s. I am also still interested in certain aspects of Anderson localization. In fact, the problem I was interested in a few years ago, has been to understand how disorder coupled to the spin degree of freedom of an electron can lead localization. I think there are lots of interesting questions in the field of transport theory. Transport problems tend to be very difficult – but I think they are not impossibly difficult; transport theory is an area where, I would expect, one might be able to make substantial progress. I think, the Boltzmann equation offers quite a few wonderful challenges for young people. All of transport theory, I think, is full of very interesting questions. – More recently, I decided that I should find out if I actually understand the physics I used to teach to my students when I still was an active professor. Another area that I also worked on in concerns the foundations of thermodynamics. A typical problem is to unravel how the second law of thermodynamics can be derived from more fundamental principles of statistical mechanics. Another one is to attempt to understand the zeroth law. The zeroth law is the deepest law of thermodynamics, and it is not well understood yet. There is renewed interest in this law that has surfaced in the wake of a new wave of interest in "many-body localization". Many-body localization – if correct – would invalidate the zeroth law for a certain class of physical systems. If you accept the zeroth law as a fundamental hypothesis, then, it turns out, the first and the second law of thermodynamics become theorems of statistical mechanics; they do not have to be postulated independently – a fact that I find quite interesting. – Well, these are some of the many areas where mathematical physics can make important contributions; one can do honest technical work on the questions arising in these areas. Another circle of problems of considerable interest to me at the moment, which, however, cannot be studied in the form of theorems and proofs, concerns cosmology. I tend to think, there must be a common mechanism underlying the following observational findings: first, there is a basic asymmetry between matter and antimatter in the universe; second, the universe shows accelerated expansion caused by something people call dark energy, the nature of which remains very enigmatic; third, data gathered from rotation curves of galaxies and gravitational lensing indicate that something called dark matter must exist; fourth, there are tiny, but highly homogeneous magnetic fields extending over intergalactic distances; (and, fifth, neutrinos are massive). My expectation – and it is a highly conjectural one, and I don't have any really solid results yet – is that these four or five facts must have a common basis; and I'm trying to find out something relevant about it.

- In your view, where does mathematical physics stand in relation to mathematics and physics? How does it relate to other sciences?

The optimistic view is the one Arthur Jaffe always likes to advertise: he likes to say that mathematical physics is the union of mathematics and physics. Unfortunately, empirically, this is not the case. Apart from this, there is the following problem: I think, many of the mathematical methods that have been developed by mathematical physicists and other people could also be used in areas not related to physics, for example in certain areas of biology and of engineering, or of information science, and so on. One may therefore expect that mathematical physics will become a broader and less sharply-defined field. The Germans already have a Max Plank Institute in Leipzig with the name "Mathematics in the Natural Sciences", and that's, I think, how the role of mathematical physics will evolve. It will be a science connecting rather diverse directions in the natural sciences and supplying natural and engineering sciences with good mathematical methods. And if such purposes are pursued seriously, I think it will become very successful again. One of the problems to overcome is that you have to learn how to communicate with people who might, for example, understand only very little mathematics and, yet, may have good problems that you could attack with mathematical methods. But communication across boundaries of different disciplines is not an easy thing; it takes much time and patience to do that. If you have a really good education in mathematics and physics, and, nowadays, in using computers in a sort of sophisticated and intelligent way, you will be able to make yourself useful in many teams and diverse fields: from mathematical finance to problems of engineering and biology, and so on. So, mathematical physics is an excellent education and training.

- Having worked in Switzerland, France and the US, could you relate to us your experience?

My stays in these countries happened during different periods of my career and life; I could not say that one was better than another one. For example, being at Princeton is not neces-

You have to learn how to communicate with people who might understand only very little mathematics and, yet, may have good problems that you could attack with mathematical methods. But communcation across disciplines is not an easy thing.

sarily better than being in Geneva or working at ETH. A job at Princeton may of course sound more prestigious; and, unfortunately, the scientific community is an extremely snobby community, and it plays a certain role in your career whether you have been at a prestigious place or not. In this respect, my stay at Princeton during my young years was certainly very useful. In fact, I have to say that I was very lucky to have been offered a good job at Princeton University. My stage of becoming a mature scientist could have happened somewhere else, too; but I was very lucky, early in my career: at Princeton, I was supported by influential people, and I met several colleagues, roughly my age, whom I had very successful collaborations with and who would become my friends. Repeatedly in my career, I have succeeded in carrying out scientific work of good quality thanks to the essential help of good collaborators without whose efforts that work would never have been completed. In France, I lived through a very productive and extremely enjoyable period of my life. I was a professor at the IHES at Bures-sur-Yvette between the beginning of 1978 and fall of 1982. This was a period when the French were surprisingly optimistic, much more so than the Germans (this has changed in the meantime).
At IHES, there was a very stimulating, optimistic and joyful atmosphere.
It was created by the people who were working there. David Ruelle and Pierre Deligne were kind and pleasant colleagues. And there was Dennis Sullivan, who was extremely interactive, talking to everybody. But he sometimes invited visitors, and, after two weeks, he decided that they were not very good, and then he lost interest in them. In some instances we had to rescue them to prevent them from falling into a depression. But, most of the time, I greatly enjoyed his company. David Ruelle became a very good friend of mine. Well, this was just a wonderful period, and I still maintain contact to the IHES. My best work with Tom Spencer was completed when I was already working at IHES. In fact, I kept in close touch with some of my American friends. The IHES is a pure research institute, which is some kind of international enclave in France. Some of my French colleagues were a little jealous of my situation and were not exquisitely friendly towards me. This is, perhaps, connected with a more general problem of life in the Paris area: this is a large area accommodating lots of bright people most of whom are busy trying to develop their own profile. Thus, they can be somewhat unfriendly towards each other. But this didn't affect me much.
After four and a half years, I left the IHES to return to Switzerland. One might ask, why I did not stay in France and enjoy my dream job. Well, I never thought that I was one of the really great guys in science, and it therefore appeared to me that spending all my life at a pure research institute might not be a good idea. But I had a very hard time deciding whether to stay in Switzerland or return to France. For quite a few years, I considered returning to the IHES. This was actually a realistic possibility for some years. But, in the end, perhaps because of inertia, I stayed in Zurich. In hindsight, I think I made the right decision! Teaching is a very useful and enjoyable activity. I learned a lot of physics by teaching it to my students. And I always much enjoyed interactions with young people, with PhD students, of which ETH always had plenty of very talented ones. I feel very privileged to have had a very good life there.

During the Mirimanoff Lectures. Photo: Maria Podkopaeva

- For a researcher, which system is more efficient: research institutes with no teaching load or the university system with some balance between teaching and research?

The answer depends on the age of the researcher in question. For most very young people, it is probably better to work in a university environment where they have many colleagues. They can compare their performance with the one of their colleagues; they can initiate collaborations, or learn something together at working seminars, and from each other, etc. But then there may come a phase where you muster a lot of creativity, and the internal pressure to realise your creative potential is considerable. During such a phase, a research institute like the IHES is, of course, the ideal place. For most scientists, highly creative periods tend to become somewhat rare at a certain age; for physicists, this tends to happen earlier than for mathematicians. It is not terribly common that physicists over the age of fifty make really significant contributions; in the world of mathematics this happens more often. But creativity simply tends to diminish with advancing age. Apart from this, I think, it is wonderful to teach. You can make yourself very useful in giving good advice to young people, guiding them in their first steps in research. So I think that research institutes are good places for people between the age of, say, 28 years and the age 40 or 45 years. Most scientists can only justify their existence (if they feel the urge of justifying their existence, which is not compulsory) by doing really good teaching. I think, our main mission is to educate young people, to train them in independent, critical thinking, to transmit to them really useful knowledge and to show them how various tools are used in practice and how one may apply what one has learned. That's our main job! Our research efforts are there primarily in order to keep us active, curious and open-minded and confronted with what is considered to be interesting and important in science, to the benefit of our educational activities. Thus, I think it is probably better for most scientists to be in a university environment than at pure research institutions – at least for scientists in the theoretical sciences.

- What was your most exciting or memorable scientific discovery and how did it happen?

That's not an easy question to answer honestly. I very much liked the problem I was working on for my PhD, namely the infrared problem in a simplified model of quantum electrodynamics. Nowadays this may appear to be a rather old-fashioned problem. But I found it extremely fascinating. Afterwards, at Princeton, I worked on two-dimensional Coulomb gases. My results had something to do with the Coulomb gas representation used in Conformal Field Theory – it was a precursor of that representation. During the same period I also successfully worked on soliton quantization.
I proved a result on φ^4 theory; namely that φ^4 theory converges to a free field theory in the continuum limit, in dimension 4 or more. The result in dimension 4 hinges on some assumption that is not understood entirely. But my results is a good result anyway; and in dimensions above 4 it is a solid theorem. My result had an unusual genesis: I was interested in the problem for quite a while. All of a sudden, during a night, I think it was after a dinner party, I woke up and I saw the right picture; and everything fell into place. But, before that, I had worked with Tom Spencer on the so-called Kosterlitz-Thouless transition in the two-dimensional rotor model and a model of random interfaces. Later we analysed the phenomenon of Anderson localization. My papers with Tom are rather excellent papers. Their genesis was entirely different from the genesis of the paper on triviality of φ^4 theory. The analysis of the Kosterlitz-Thouless transition and the one of Anderson localization required very long periods of hard work. The φ^4 story and the Coulomb gas were basically understood in a single night, which has been quite an exhilarating experience.
My work on soliton quantization led to the implicit discovery of braid group statistics, which I started to study explicitly more than ten years later, in 1987. Braid group statistics for quantum fields first appeared in work that has been largely forgotten: on one hand, in work due to Ray Streater and Ivan Wilde, and, on the

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other hand, in my work on soliton quantization in two-dimensional scalar quantum field models. In my opinion, these papers were the starting point on the road to the discovery of braid group statistics. Unfortunately, I didn't know what a braid group was yet when I wrote my papers on soliton quantization in 1975. The work focussing on braid group statistics started in 1987. I enjoyed a sabbatical, back at the IHES, together with Vaughan Jones. He is an incredibly friendly and generous colleague. I was working on a problem motivated by the quantum Hall effect; it involved braiding Wilson lines with each other. I asked Vaughan a number of technical questions, and he explained to me what a braid group is, and many other things, such as a Yang-Baxter matrix, and so on. After I had listened to him, everything became clear. I wrote several papers about braid group statistics in two-dimensional quantum field theory and three-dimensional gauge theories. I consider that work to be among my most interesting contributions to physics. During every period of my professional life, I felt that there were exciting problems around and great discoveries to be made. But I do not consider myself to be one of the great scientists – I just enjoy doing science. Some of my work is decent; but if I had not done it, the world wouldn't look different – or somebody else would have done it.

- Have any of your hobbies helped you in your research? What do you do outside of work that might inspire you?

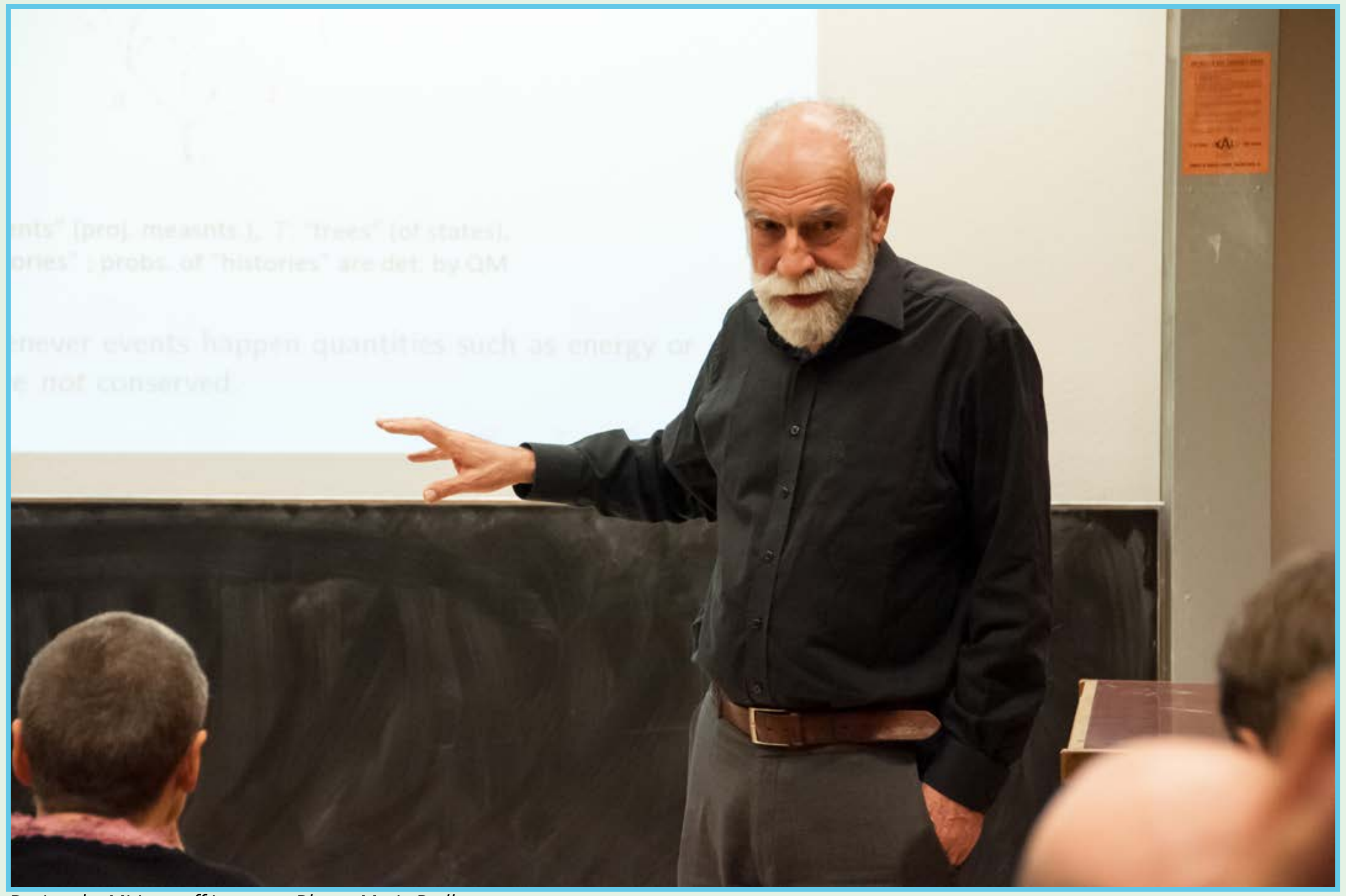
I'm a very passionate hiker. I also used to do skiing in my younger years. I do a little swimming. These physical activities are, I think, very important for me. I have to feel comfortable physically: otherwise I cannot think very well. Another activity that I used to engage in (but unfortunately not in recent times) and that I used to like a lot is drawing and painting. Furthermore, I have been a fairly productive writer of little essays. Some of them were not intended to be distributed widely, others were published – some for example in the ETH online journal, as columns. I like to write such essays. I also like to dance; but my wife doesn't like it much; so I don't have many opportunities to dance, anymore. But I still enjoy it.

During every period of my professional life, I felt that there were exciting problems around and great discoveries to be made. But I do not consider myself to be one of the great scientists – I just enjoy doing science.

- In your manifesto, you say that scientists should take on a greater role in the political and social life of our planet. Could you comment on your position?

There is a very strange phenomenon. Scientists, in particular scientists interested in mathematics or the natural sciences, are not considered to be intellectuals anymore. Intellectuals are either poets or composers or philosophers... But I think, we physicists and mathematicians have very non-trivial ideas about Nature, the Universe, Humanity and society, and so on. And we have been trained to think clearly and logically. You know, many people who have an education in the humanities are quite confused about what it means to have a clear thought. I therefore think we have a big responsibility to introduce clear thinking into society. We also have the duty to express our opinions. After all, we are citizens of a country, we are human beings; and thus we have to try to make ourselves heard. Moreover, I think it feels good to be engaged in some meaningful activity and to bear responsibility. I am alarmed by my hopefully mistaken impression that many young people have resigned, are by and large inactive politically, and are not engaged in anything useful for society. I think this is a big mistake. At present, we are in a very precarious, dangerous situation. If we do not take clear positions and are not involved in improving it, then various crazy politicians might do things that we will not approve of at all. Thus, I think it is a necessity that we make ourselves heard and understood, that we participate in political debates etc… I think modern life has very major shortcomings. We have become slaves of, for example, the media; we always have to read email; we have to inspect our cell phone every few seconds; we have to engage in often somewhat silly outreach activities, which are sometimes quite dishonest, because they convey a picture of science that is too romantic;
and then there is all the fuss about quality control, evaluations, and so on. We have become slaves of a system that "castrates" us as political actors. I find all this quite worrisome, and I am appalled by many trends of modern life. By nature, I am in a sense an anarchist. Of course, I enjoy my quiet, pleasant life. But I certainly do not appreciate the prevailing political inactivity and the feeling that we cannot change anything for the better anymore, that the big disaster cannot be avoided anymore. This is a defeatist attitude that I find extremely unappealing, and which upsets me. The fact that there are wars in Europe again is a terrible thing for me. Somehow it is simply accepted that, in the Ukraine, people kill each other again. I think this is actually totally unacceptable. Another fact, I feel, is extremely bad: that there is

I think we have a big responsibility to introduce clear thinking into society.



During the Mirimanoff Lectures. Photo: Maria Podkopaeva

no serious, open debate about how we can build a society where Christians and Muslims, Buddhists, people from different cultural backgrounds will peacefully live together, in mutual respect and to each other's profit and enrichment. The role and status of women must be rethought. Our ideas of the dignity and the rights of women are not shared by some of the people who now arrive in Europe. It should be debated whether we accept it that some people in our society have a picture of women and their rights and duties that is totally different from ours. I feel we must openly address topics that divide us and find out whether we can reach a common ground. Otherwise, parallel societies will form. This is happening in France: there are areas in the suburbs of Paris where a parallel society of people exists who absolutely don't want to become French or subscribe to traditional values of the French republic. This is not good; I think it is actually very dangerous in the long run. Young people should debate such problems and deal with them, and become more active in attempting to solve them. I have shown my manifesto at the end of quite a few of my lectures, and it has unfortunately been very rare that it triggered some discussion. Who knows why that is; I haven't given up my hopes that things will change for the better yet. Incidentally, I will organize a "summer academy" about these questions for students of the so-called "Studienstiftung", which is a foundation supporting talented students. After high school, young people can apply to be admitted as members of the "Studienstiftung"; and I will organize a program for such students in the fall of 2016. This will be an experiment, and I hope it will be successful!

Conversation with Jürg Fröhlich
December 2015, Geneva

Interviewed by Maria Podkopaeva
Science Officer, NCCR SwissMAP

For the full text please visit the SwissMAP website:
http://nccr-swissmap.ch/articles/conversation-jurg-frohlich

Presenting the SwissMAP

Master Class
in Planar Statistical Physics

2015-16

The aim of the SwissMAP Master Class in Planar Statistical Physics 2015/16 is to provide a small number of outstanding students with Master-level courses in probability together with more advanced courses in the field of planar statistical physics.

Amazing progress has been made in the understanding of planar statistical physics during recent years. The introduction of the Schramm-Loewner Evolution and the developments in the theory of random planar maps and lattice models enabled mathematicians to connect the rigorous approach to statistical physics with the traditional approach from physics based on exact integrability and conformal field theory. The goal of this program is to provide courses on recent advancements in this field. More general courses on probability will also be provided during the first term.

The program is aimed at Master students, though advanced undergraduates and beginning PhD's are also welcome. The participants enroll in a one-year master program at the university of Geneva, providing 60 ECTS credits. If they wish, participants will be offered the possibility of finishing a master degree from the university of Geneva by completing a Master thesis for 30 additional ECTS credits.

The program started in September 2015 and will be completed in June 2016. 12 students from 8 different countries are enrolled in this Master Class alongside local students who were invited to attend the lectures.

On the following pages you will find an introduction to the lecturers and the chosen students.

Next year's Master Class is already in the making and will be focusing on geometry, topology and physics.

The Lecturers & Courses

Dmitry Chelkak

Brownian Motion and stochastic calculus

Prof. Chelkak is currently a visiting professor at the University of Geneva.
He received the Salem Prize in 2014. His research interests include conformal invariance in critical 2D lattice models, discrete complex analysis and spectral theory.

David Cimasoni

On various aspects of the dimer and planar Ising models

David Cimasoni is a senior lecturer at the University of Geneva. His research interests deal with invariants of knots and links in all their forms, especially classical invariants such as the Alexander polynomial and the Levine-Tristram signature.

Nicolas Curien

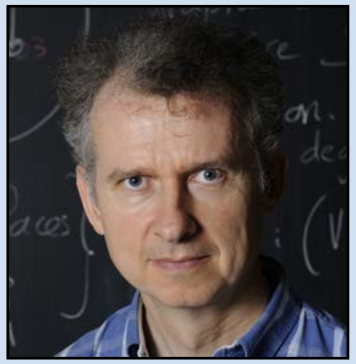
Random planar maps - Part 1

Prof. Curien is a professor at the University Paris-Sud Orsay. He received the Rollo Davidson prize in 2015. He studies asymptotic geometric properties of large random combinatorial objects, in particular, the scaling limits of random maps and trees.

Hugo Duminil-Copin

Geometric representations of lattice models

Prof. Duminil-Copin is a professor of mathematics at the University of Geneva. He received the IAMP Early Career Award in 2015. His research deals with mathematical physics, probability, complex analysis and combinatorics.



Jean-Francois Le Gall

Random planar maps - Part 2

Prof. Le Gall is a professor at the University Paris-Sud Orsay. His research interests include probability theory, Brownian motion, Lévy processes, superprocesses and their connections with PDE, Brownian snake, random trees, branching processes, stochastic coalescence and random planar maps.

Stanislav Smirnov

Conformal invariance of lattice models

Prof. Smirnov is a professor of mathematics working at the University of Geneva. He was awarded the Fields Medal in 2010. His research focuses on the fields of complex analysis, dynamical systems and probability theory.

Yvan Velenik

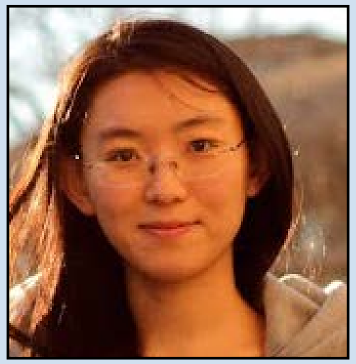
Introduction to statistical mechanics

Prof. Velenik is a professor of mathematics at the University of Geneva. His research interests lie mainly in the applications of probability theory to the study of rigorous classical statistical mechanics, especially lattice random fields and random walks.

Wendelin Werner

Schramm-Loewner Evolution and Gaussian Free Field

Prof. Werner is a professor of mathematics at the ETHZ in Zurich. He was awarded the Fields Medal in 2006. His research focuses on random processes such as self-avoiding random walks, Brownian motion and Schramm-Loewner evolution



Hao Wu

Martingales and Markov processes

Hao Wu is a postdoc researcher at the University of Geneva. Her research interests include: Schramm Loewner Evolution, Conformal Loop Ensemble, Gaussian Free Field, percolation and the Random Cluster Model/Potts Model.

Full list of courses:

First semester courses:

1. Introduction to statistical mechanics *(Y. Velenik)*

2. Brownian Motion and stochastic calculus *(D. Chelkak)*

3. Martingales and Markov processes *(H. Wu)*

4. Geometric representations of lattice models *(H. Duminil-Copin)*

5. Schramm-Loewner Evolution and Gaussian Free Field *(W. Werner)*

Second semester courses:

6. Conformal invariance of lattice models *(S. Smirnov)*

7. Random planar maps *(N. Curien, J-F Le Gall)*

8. On various aspects of the dimer and planar Ising models *(D. Cimasoni)*

12. Winter School in Les Diablerets with talks by: Jean-François Le Gall, Nicolas Curien, Itai Benjamini, Christophe Garban, Ilia Binder and Clément Hongler.

All classes held in UNIGE can be viewed on the MediaServer: https://mediaserver.unige.ch/collection/VN3-222c-2015-2016

The Students

Some of the SwissMAP students alongside Prof. Werner and UNIGE / ETHZ students, during the Zurich class. Photo: Christina Buchmann

The first SwissMAP Master Class 2015-16 welcomed 12 international students, hand-picked by our committee based on their academic skills, knowledge and achievements. Ten of the students take part in the complete program, and two more joined for the second, more advanced, semester.

The students come from a large variety of countries: Brazil, Canada, Chile, Finland, France, Italy, Russia and the USA.

Here are their stories:

Giovanni Antinucci:
Giovanni comes from Italy and is currently a PhD student in mathematics at the University of Rome. He received the cum laude distinction for his secondary school, bachelor's and master's diplomas. He received the Excellent Graduate Prize in 2012-13. In his PhD work, he is studying the problem of the construction of the ground state of a one-dimensional system of interacting spinless fermions with Dirichlet Coundary Conditions using renormalization group methods.

Mikhail Basok:
Mikhail was born in Russia. He studied at the Saint Petersburg State University and is currently a PhD student at the Steklov Institute for Mathematics in St. Petersburg and an assistant at the Chebyshev Laboratory. Mikhail is interested in moduli spaces and their links with loop ensembles.

Hannah Cairns:
Hannah comes from the USA. She graduated at the University of British Columbia and is currently a PhD student in Mathematics at the Cornell University. As a student, she received several distinctions including the G C Webber Memorial Prize, the Lorraine Schwartz Prize in Statistics and Probability and the Reginald Palliser-Wilson Scholarship. Among her research interests are applications of probability to analysis.

Danila Cherkashin:
Danila was born in Russia and is currently a student in mathematics of the Saint-Petersburg State University. Danila is interested in combinatorics of hypergraphs.

Leticià Dias Mattos:
Leticià comes from Brazil and obtained her Master's degree at Universidade Federal de Minas Gerais. She received second place in the International Mathematics Competition for University Students 2014.

Luis Fredes Carrasco:
Luis was born in Chile and received his Master's degree in mathematical engineering and operation research at the University of Chile. As a student, he participated in several internships on facial recognition software and stochastic optimization and received outstanding student awards.

Alex Karrila:
Alex comes from Finland and has recently started his PhD at the Aalto University in Helsinki. In his research and studies, Alex is interested statistical physics and more generally, in mathematical problems motivated by physics and engineering.

Dmitrii Krachun:
Dmitrii comes from Russia and is currently an undergraduate student at the Saint Petersburg State University. He received gold medals of the International Mathematics Olympiad in 2012 and 2013, won multiple prizes at other Olympiads and math competitions.

Ekaterina Mukoseeva:
Ekaterina was born in Russia and studied mathematics at the Saint Petersburg State University and is an assistant at the Chebyshev Laboratory. Ekaterina is a multiple prize winner of the Russian Olympiad in Mathematics since 2009. Apart from the subject of the master class, Ekaterina is interested in the theory of differential equations.

Shalin Parekh:
Shalin comes from the USA. He graduated from the Stony Brook University. There, he held a presidential scholarship, was part of the Honour's College and was on the Dean's List for all completed semesters.

Larissa Richards:
Larissa was born in Canada and is currently a PhD student in mathematics at the University of Toronto. She is also a teaching assistant and has earned 4 scholarships between 2011 and 2014. Larissa's research is focused on stochastic processes.

The Zurich class. Photo: Christina Buchmann

Maud Szusterman:
Maud comes from France and holds a Master's degree in probability from the ENS Lyon. Her research interests revolve mainly around probability, in particular all kinds of random walks, and discrete objects in planar geometry (random or not).

Prof. Wendelin Werner during the Zurich class. Photo: Christina Buchmann

The 2nd Master Class: Geometry, Topology and Physics 2016 – 2017

The 2016-2017 master class covers some of the most important and actively developed subjects of the research area in-between geometry, topology and physics providing an entry point into the forefront research for students starting to work in this field. The program is aimed at Master students and beginning PhD students.

The master class will include the following courses:

- Symplectic geometry of moment maps – *A. Alekseev*
- Poisson geometry and quantization – *P. Ševera*
- Topological aspects of algebraic geometry – *I. Itenberg and G. Mikhalkin*
- Symmetries and moduli spaces – *S. Galkin and A. Szenes*
- Introduction to quantum topology – *R. Kashaev and A. Virelizier*
- Quantum mechanics for mathematicians – *M. Mariño*
- Field theory for mathematicians – *A. Alekseev*

Events 2016

Past Events:
Fall 2014 - Spring 2016
The first SwissMAP year held within itself a great number of conferences and events.
Some highlights include, but are not limited to: the Higher Structures 2014 conference which was held in Geneva; The 1st, 2nd and 3rd Geometry and Topology conferences (the 4th one is already in the making); the Swiss Knots 2015 conference in Geneva; two SwissMAP General Meetings during which the annual SwissMAP Innovation Prize is presented; two Doctoral Schools spread out across Amsterdam, Brussels, Paris and Geneva; the Mirimanoff Lectures during which we sat down with Jürg Fröhlich; the Complex Analysis conference held in Saas-Fee, and of course, the two series of public conferences that were held in Geneva and Zurich celebrating 100 years of General Relativity.

For the complete list, please visit:
http://nccr-swissmap.ch/events

June

02 – 03 June
2nd SwissMAP Site Visit
ETHZ, Zurich
The 2nd SwissMAP Site Visit will be in Zurich.

19 – 24 June
Integrable Systems - Conference
CSF, Ascona
The Integrable Systems conference will be held at the Congressi Stefano Franscini (CSF) in Ascona.

27 June – 02 July
Poisson 2016 - Summer School
UNIGE, Geneva
The summer school is aimed at students in their final years, PhD students and young researchers. All lectures will take place at the Sciences II building, room A300 (24 quai Ernest Ansermet).

July

04 – 07 July
Poisson 2016 - Conference
ETHZ, Zurich
The conference will be attended by students and researches from around the world, including some of the leading mathematicians in the field of Poisson Geometry.

Aug – Sep

22 – 26 August
RAQIS'16 - Conference
UNIGE, Geneva
RAQIS (Recent Advances in Quantum Integrable Systems) is part of the RAQIS series and is a satellite conference of STAT-PHYS 26.

11 – 14 September
SwissMAP General Meeting
Engelberg
The 3rd SwissMAP General Meeting will take place in Engelberg. The SwissMAP Innovator prize 2016 will be announced as part of the event.

01 Sep – 30 June
Master Class in GTP - School
UNIGE, Geneva
Geometry, Topology and Physics SwissMAP project organizes a year-long master class at University of Geneva in the academic year 2016/2017 for master and beginning PhD students.

Oct – Nov

28 Nov – 16 Dec
Amsterdam-Brussels-Geneva-Paris Doctoral School
UNIGE, Geneva
The 2016 String Doctoral School will be held in Brussels (3 - 21 October); Amsterdam (31 October - 18 November) and Geneva (28 November - 16 December).

Dec – Jan 2017

09 – 13 January
Winter School in Mathematical Physics
Les Diablerets
The annual Winter School in Mathematical Physics will take place in Les Diablerets from January 09 to January 13, 2017.

16 – 20 January
4th Geometry and Topology - Conference
Engelberg
This conference is the fourth in the series of SwissMAP Geometry and Topology conferences and will take place on January 16 - 20, 2017 in Engelberg.

New Collaborators


C. Keller


J. Sonner

Christoph Keller obtained his PhD with Matthias Gaberdiel at ETH Zurich in 2008. After that he spent a year as a postdoc at Harvard, three years as a McCone fellow at Caltech, and three years as a postdoctoral researcher at Rutgers University. In 2015 he was hired as a SwissMAP assistant professor at ETHZ (mathematics department). Christoph's research lies mainly in the mathematical structure of quantum field theory and string theory. He is interested in conformal field theories and their applications to physics and mathematics.

Julian Sonner obtained his PhD in theoretical physics at the University of Cambridge under the supervision of Paul Townsend (FRS) and remained there after graduation as a Fellow of Trinity College splitting his time between DAMTP in Cambridge and the Theory Group of Imperial College in London. Afterwards he spent three years as a postdoc at MIT. In 2015 he was hired as an associate professor at the University of Geneva (physics department). Julian's research focuses on the applications of holographic methods to condensed matter physics, and in particular to non-equilibrium problems. He is also interested on supergravity, quantum gravity, and general aspects of the AdS/CFT correspondence.


I. Manolescu


A. Knowles

Ioan Manolescu was a student of the ENS Paris and has obtained his PhD from the University of Cambridge under the supervision of Geoffrey Grimmett in 2012. Afterwards he was a postdoc at the University of Geneva. He was hired as an associate professor at the University of Fribourg, thus enlarging the SwissMAP network. Ioan's research interests lie in probability, more precisely in problems inspired by statistical mechanics. He works with percolation, the random-cluster and Potts models, and self-avoiding walk.

Antti Knowles obtained his PhD in theoretical physics at ETH Zurich under the supervision of Jürg Fröhlich and then spent 3 years as a postdoc at Harvard. After that he has successively been an assistant professor and Courant Instructor at the Courant Institute, New York, a member of the Institute of Advanced Studies, Princeton and an assistant professor at ETH Zurich. In 2016 he was hired as a tenure track assistant professor at the University of Geneva. Antti's research lies in the areas of probability, analysis and mathematical physics. More specifically, he is interested in random matrices, random graphs, statistical mechanics, stochastic processes, high-dimensional statistics and quantum dynamics.

Marcello Porta did his PhD with Giovanni Gallavotti and Vieri Mastropietro at the University of Rome "La Sapienza". After that he has been a postdoc at ETH Zurich and the Universities of Bonn and Zurich. In 2016 he was hired as a SwissMAP assistant professor at the University of Zurich. Marcello works in quantum many body theory and condensed matter physics.

João Penedones obtained his PhD at the University of Porto in 2007 under the supervision of Miguel Sousa da Costa. He has afterwards spent several years at Kavli Institute for Theoretical Physics and Perimeter Institute for Theoretical Physics before returning to the University of Porto as a research associate. In 2016 he was hired as a SwissMAP assistant professor (tenure track) at EPFL. João works on conformal field theory, gauge-gravity duality and string theory.


M. Porta


J. Penedones

Awards

Hugo Duminil-Copin

IAMP Early Career Award

Congratulations to Hugo Duminil-Copin for receiving the IAMP Early Career Award for his fundamental contributions to the rigorous analysis of critical phenomena, in particular for establishing the critical point in the two-dimensional random cluster model and for the proof of the continuity of the magnetization at the critical temperature in the ferromagnetic Ising model.

Prof. Duminil-Copin is a professor of mathematics at the University of Geneva. His research deals with mathematical physics, probability, complex analysis and combinatorics.

Tudor Ratiu

ARA Excellence in Sciences Award

Congratulations to Tudor Ratiu who received the American-Romanian Academy of Arts and Sciences (ARA) Award of Excellence in Sciences.

The award was announced during the 39th Congress of the American-Romanian Academy of Arts and Sciences in July 2015.

Tudor Ratiu is an Honorary Professor at EPFL. His research interests center on geometric mechanics, symplectic geometry, global analysis and infinite dimensional Lie theory, together with their applications to integrable systems, nonlinear dynamics, continuum mechanics, plasma physics, and bifurcation theory.

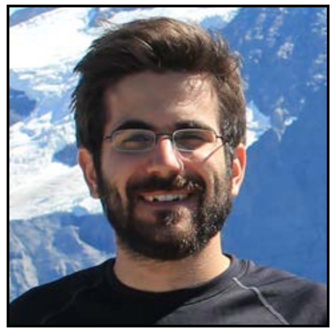
Grants

Thomas Willwacher

ERC Starting Grant

ETH, Zurich

Congratulations to Thomas Willwacher who received an ERC Starting Grant entitled *A graph complex valued field theory.* The goal of the project is to study classical, little understood objects in algebraic topology (such as configuration spaces, embedding spaces and diffeomorphism groups) through spaces Feynman diagrams, equipped with suitable algebraic structures.



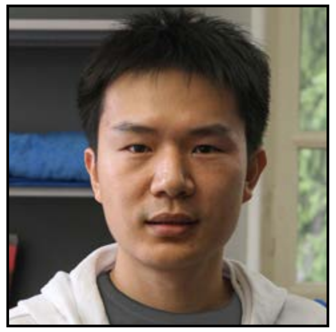
Alexander Glazman

Early Postdoc.Mobility Fellowship

Tel-Aviv, Israel

Congratulations to Alexander Glazman for obtaining the Fellowship for the project *Observables on lattice models* concerning discrete models of statistical mechanics, such as the Ising model and the self-avoiding walk. One of the objectives in this project is to investigate the underlying combinatorial structure of the models, try to make use of recently discovered observables for the loop O(n) models satisfying a part of discrete Cauchy-Riemann relations.

Xiaomeng Xu



Early Postdoc.Mobility Fellowship

MIT, Boston USA

Congratulations to Xiaomeng Xu for obtaining the Fellowship for the project *Differential equations with irregular types and Drinfeld twists.* This project lies at the crossroads between mathematical physics, geometry, algebra and analysis. Its purpose is to establish a deep connection between several very different modern mathematical subjects: the quantization problem in Poisson geometry and the deformation problem of certain differential equations with irregular singularities.

Puzzle Corner

1. Crossing the Geneva lake

The distance between Evian and Lausanne Ouchy is 12.5 km. Due to the roundness of the Earth, if a swimmer wants to swim in a straight line, he must swim underwater. What is the maximal depth he will reach?

2. Arthur and the Knights of the Round Table

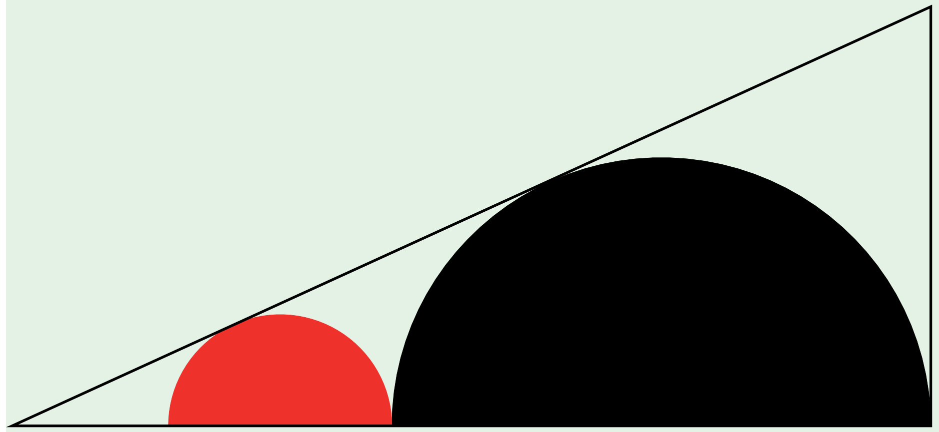
King Arthur and nine knights are seated around the Round Table. The king has a pile of ten plates in front of him and the knights do not have any plates. In one step, a person having two plates is authorized to give one of his plates to his left neighbor and, at the same time, another of his plates to his right neighbor. Can we get to a situation where everyone has exactly one plate?

3. Sangaku

We denote by h the altitude from the base of the right triangle in the picture, R the radius of the black half-circle and by r the radius of red half-circle. Express the ratio r/R in terms of R and h.

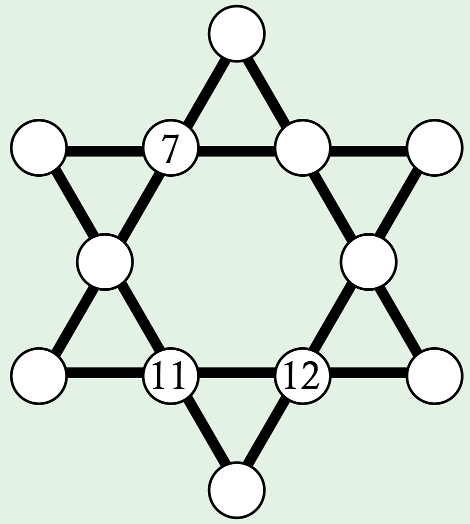
Hint : The radius R of an inscribed circle in a triangle is given by R = 2S/p where S is the area of the triangle and p its perimeter.

From Sangaku. Le mystère des énigmes géométriques japonaises, Géry Huvent, Dunod, 2008



4. Hexagram

Fill the hexagram with numbers from 1 to 12 so that the sum in each direction is always 26.



5. The Number 24

Obtain the number 24 using only the numbers 1, 3, 4 and 6, arithmetic operations (addition, subtraction, multiplication and division) and parentheses. Each number can only be used once. Operations and parentheses can be used as many times as you want. It is not allowed to combine numbers (e.g. use 1 and 3 to get 13).

6. The Card Trick

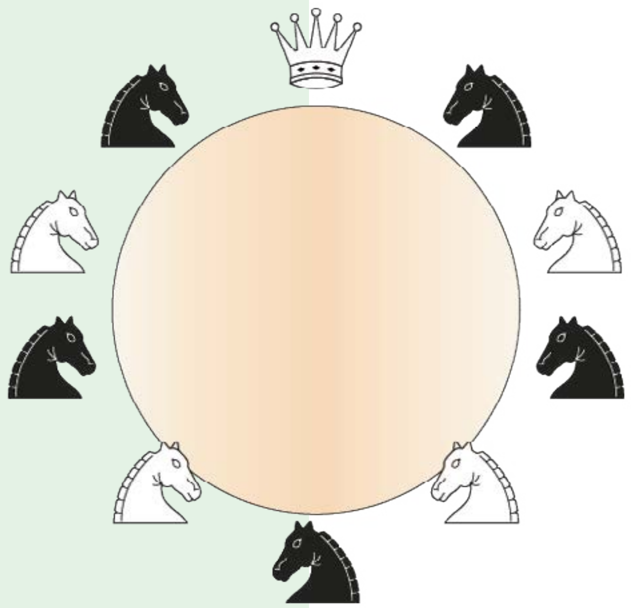
A magician and an assistant perform the following card trick with a deck of 10 cards. A member of the audience shuffles the cards, shows them just to the assistant, and places them face down in a row on a table. Then the assistant turns up 6 of the cards on his own choice, by going either left-right or right-left along the row.

Then there are 4 cards face-down remaining, and the magician correctly identifies each one of them. How can they agree on a strategy for this trick?

Answers

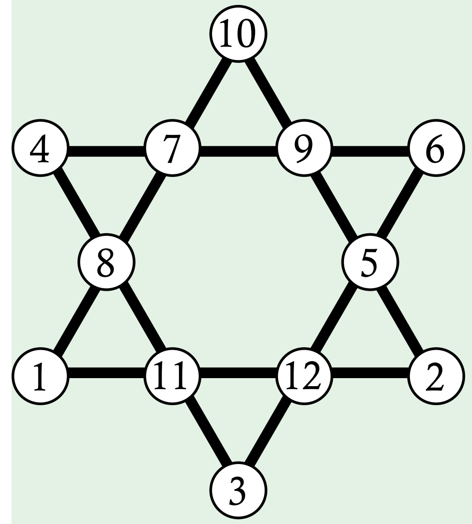
1. Crossing the Geneva lake
The maximal depth the swimmer reaches is about 3 meters.

2. Arthur and the Knights of the Round Table
It is not possible to distribute the plates so that each knight has one plate in front of him. Indeed, separate the knights into two groups as shown in the picture. Note that every time a knight of the white group distributes two plates, two plates arrive in the hands of a knight of the black group. In other words, during each movement, an even number of plates is transferred from a group to another group. However, each group consists of five knights. Since King Arthur has 10 plates (an even number of plates) and since 5 is not even, it is impossible to distribute the plates so that each knight has one plate in front of him.



3. Sangaku
The ratio is : $r / R = R^2 / h^2$

4. Hexagram



5. The Number 24
$6/(1-3/4) = 24$

6. The Card Trick
This card trick (see [2]) was discussed in a session of the ETH Math Youth Academy, illustrating the Pigeonhole principle. It is based on the case n = 3 of the following theorem of Erdos and Szekeres (see, for example, [1], p. 162):

Theorem 1. Any sequence of $n^2 + 1$ pairwise distinct real numbers contains a monotonic subsequence of length $n + 1$.

Proof. Consider a sequence $a_1, \ldots, a_{n^2+1}$ of pairwise distinct real numbers. For each $i = 1, \ldots, n^2 + 1$; denote respectively by x_i and y_i the lengths of the largest increasing and decreasing subsequences beginning at a_i. We claim that $(x_i, y_i) \neq (x_j, y_j)$ for $i \neq j$. Indeed, say $i < j$. If $a_i < a_j$, then $x_i > x_j$, since one can add a_i to an increasing subsequence of length x_j beginning at a_j. Similarly, $y_i > y_j$ if $a_i > a_j$.
If we assume that $x_i, y_i \leq n$ for all $i = 1, \ldots, n^2 + 1$; then there would be only n^2 possibilities for each pair (x_i, y_i): However, we have $n^2 + 1$ pairwise distinct such pairs, contradicting the Pigeonhole principle.

First, the magician and the assistant agree on an ordering of the 10 cards. The assistant finds a monotonic subsequence of 4 cards and simply turns up the other 6 cards (going left–right or right–left depending on whether the chosen monotonic subsequence is increasing or decreasing). The magician now knows what the 4 face–down cards are as a collection and how they are ordered, so he can easily identify each one of them.

References:
[1] M. Aigner, G. Ziegler, Proofs from the Book, Springer, 1998.
[2] B. Epstein, All you need is cards, in Puzzler's Tribute, D. Wolfe, T. Rodgers (eds), A K Peters, 2002.

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