SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

SwissMAP Perspectives

Issue 2 | 2017

Contents

In Search of Lost Information

When first learning about quantum mechanics one is introduced to the idea that its formalism is inherently non-deterministic, that one of the most mysterious properties of our best description of nature is that it cannot predict the future with certainty, but rather that only probabilities for the occurrence of physical events are knowable. This is in striking contrast to our ordinary conception of the world around us, built on the experience of classical physics, where a given initial state leads with inevitable certainty to a uniquely determined final state. This is the idea that quantum mechanics does away with determinism, the much-cherished principle of classical physics: if the initial state of a system, say an arrangement of particles, is known, then the state at any future time is also known, whence the infamous collision of classical physics and the quantum world. But not so fast! In fact, deterministic evolution has an important role to play in quantum mechanics as well. Although in this case it is the probability distribution of events, or more precisely, the quantum wave function, that evolves deterministically: if we know the initial probabilities of events, their probability at any later time follows inexorably from the laws of quantum mechanics[1].

It is thus rather surprising to learn that when we add Einstein's theory of gravity to the story, it would appear that it is quantum mechanics that is too deterministic to accommodate the bizarre behavior of black holes; the subtle effects of warped space-time around these objects seem to lead to a violation of the deterministic evolution of the wave function, without which the original quantum theory seems to make little sense. The name of this mystery is the "black-hole information loss paradox", which, posed in 1976 by Stephen Hawking, has inspired much of the subsequent research in quantum gravity. And despite being declared solved time and time again, as inevitably as a planet follows its orbit around the Sun, the information-loss debate has flared up time and time again since its original formulation, perhaps leaving us none the wiser, even today. Recent years have seen a rejuvenation of this debate in the context of string theory and holographic duality, two notions that we will explore in our search of lost information.

Gravity and warped space and time: where forces aren't forces & black holes aren't black

The general theory of relativity, i.e. Albert Einstein's classical theory of gravity, describes the gravitational force not as a force in the Newtonian sense, but as a manifestation of the curvature of space and time.

Let us neglect time on a first pass, and the idea is perfectly simple: in a curved space, such as the surface of a sphere, curves of minimal length connecting two points are not straight lines, they are segments of great circles, a phenomenon that anyone taking a flight from Europe to the US is familiar with. The reason why the Earth follows a curved orbit around the Sun, according to Einstein, is not the attractive force of Newton, but instead the manifestation of the fact that Sun's gravity has curved space around it in such a way that the shortest path chosen by Earth is no longer a straight line but, to a very good approximation, a Keplerian orbit. Were there no central star, space would be flat, and Earth would instead follow a straight line. So far this is perfectly intuitive and once couched in the mathematical language of Riemann, Einstein and Hilbert, more accurate than Newton's laws of gravity (witness the perihelion precession of Mercury).

For the sake of human intuition, we allowed ourselves an important, but ultimately inaccurate simplification by omitting time from the discussion. Once we add it back in, the notion of curved trajectories has to be carried over to a new sort of geometry, one which weaves space and time into a single object, namely Einstein's famous notion of the space time continuum. The effect of masses and energies is then to curve and bend space time, leading to many infamous predictions, among them time dilation and the existence of event horizons. The former has become a mainstay of science fiction writing, even making an appearance as a plot device in the movie "Interstellar": the crew of a spaceship operating in the vicinity of a very heavy object (actually a black hole) experiences time elapsing far more slowly. Returning to their mothership far from the hole, they find the rest of the crew literally aged beyond their years.

> **God doesn't play dice.**
>
> **- A. Einstein**

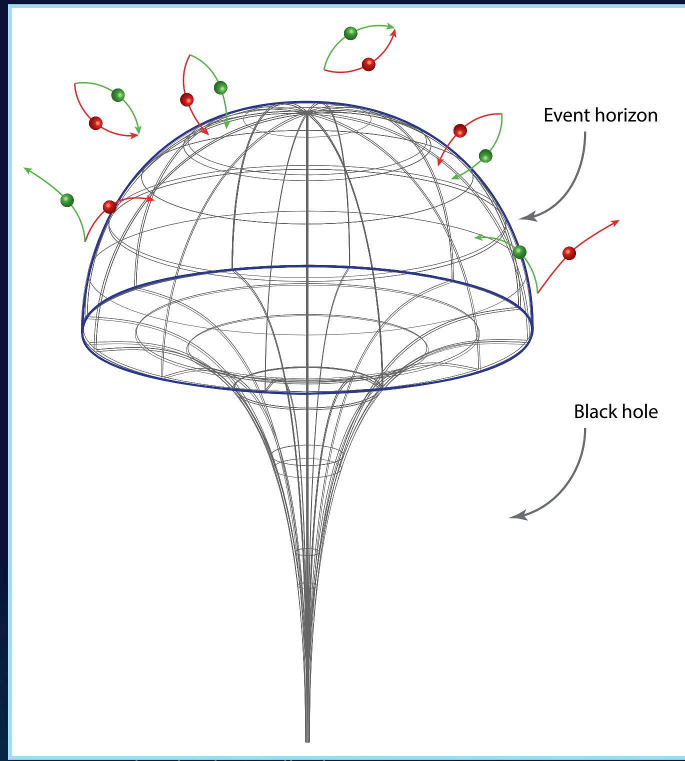


Figure 1. Credit: Shaula Fiorelli Vilmart

It is however the latter, namely event horizons, that interest us most in this essay. Space time curvature becomes more pronounced the more mass and energy is present in a given region, up to a point where even the trajectories of light are bent into such contortion that no paths exist any longer which would allow it to escape from the vicinity of the accumulation of mass. Since light is the least affected by the curvature of space time (in other words, nothing moves faster than light), no physical object can escape it either. The boundary of this region is called the event horizon, delimiting a province of space time totally shielded from the outside, its insides forever hidden from all exterior observation. To an outside observer, the region inside the event horizon is thus characterized by nothing but blackness, which famously led John Wheeler to popularize the expression "black hole" for it (see Fig. 1).

> **God not only plays dice, He sometimes throws the dice where they cannot be seen.**
>
> **- S. W. Hawking**

Black holes were discovered as early as 1915, as a consequence of General Relativity, by Karl Schwarzschild, a mere three months after Einstein published his theory of gravity, and only six months before Schwarzschild himself tragically passed away at the Russian front. Their properties, however, were so puzzling and their implications so alarming that their existence, even theoretically, was not accepted up until the 1960s and 1970s, a period that marked the golden age of black-hole research, driven by such luminaries as Jakob Bekenstein, Roger Penrose, Stephen Hawking and Roy Kerr. What this period of rapid development culminated in was nothing short of sensational and keeps many of us busy to this day. But let us trace these developments step by step.

The first important observation was made by Bekenstein. Because black holes swallow everything and everybody unfortunate enough to venture too far into their gravitational abyss, we are forced to assign an entropy to the black holes themselves, as if they were thermodynamic objects. If we did not do so, Bekenstein argued, the second law of thermodynamics could be violated when black holes swallow up their surroundings. Furthermore, this entropy has to be proportional to the area of the event horizon to be consistent with all of his Gedankenexperiments. This is a first clue that something very special is going on with black holes: consider any other known substance, and one would conclude that entropy, essentially a measure of the number of allowable configurations, should be proportional to the bulk volume of the substance. Take for example the air surrounding you right now: clearly any molecule of this gas is free to explore any configuration in three-dimensional space compatible with the boundary conditions. Accordingly, the number of possible configurations available to it should also scale with the volume, and not with the surface. But this area scaling is precisely what the configurations of a black hole are wont to do.

Why is there nothing rather than something?

Wherever there is entropy, there also is temperature. The presence

1. *Again, for reasons of accuracy, we should really talk about the initial and final wave function here.*

of temperature in turn implies that any object possessing it must emit a spectrum of particles, so-called black-body radiation, characterized by only one number, namely the temperature. But here we find ourselves at the first serious impasse of our story. Black holes and their event horizons strictly confine everything they consist of inside them, yet we concluded that to be consistent with the laws of thermodynamics, they should have a temperature. How then can this thermal radiation escape, when in fact it should not, by definition? Faced with the age-old question of being and nothing, in physics, unlike in philosophy, the answer can be found in the quantum world. Until now our entire discussion was based on classical physics, namely the theory of General Relativity. Quantum mechanics supplies the missing link, as it forces us to grapple with the very notion of Nothingness. According to the Heisenberg uncertainty principle, Nothing is impossible or, more precisely, very improbable. Instead the state of Nothingness ("the vacuum") is continuously negated by the materialization of pairs of particles and antiparticles, only to vanish again after a brief period of existence, by mutual annihilation. This dissolution of the pair restores the balance that was upset by its appearance, which, however briefly, resulted in the existence of a negative amount of energy. If this happens close to a black hole, it may occur, however, that one member of the pair is unfortunate enough to plunge into the abyss of the black hole and disappear from the outside world. Having no partner to annihilate with, the remaining member of the pair is left with no choice but to manifest itself as a particle emanating from the horizon region, leaving the black hole to foot the negative energy bill. In other words, the pair creation has two consequences: the emission of positive energy particles from the horizon region, and the loss of small amounts of energy from the black hole, with each such emission (see Fig. 1). Expressing these processes in terms of the mathematical

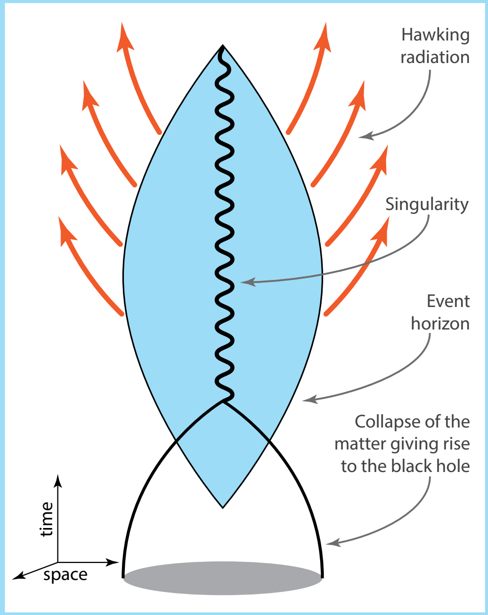


Figure 2. Credit: Shaula Fiorelli Vilmart

formalism of quantum field theory, Stephen Hawking calculated the properties of the resulting emission spectrum, leading to the spectacular realization that it exactly corresponds to the missing black-body radiation, and thus accounting for the temperature of the black hole[2]. It seems that the inevitable conclusion is that not only do black holes really possess all the qualities of a thermodynamic system (including some we have not had the time to discuss here), they also slowly evaporate into thermal radiation as time progresses. But once this process of evaporation has run its course[3], nothing is left but thermal radiation, a state that is fully captured by only a single number, the temperature[4] (see Fig. 2).

But herein exactly lies the problem: the course of events we have just described implies a breakdown of predictability so egregious that not even quantum mechanics can tolerate it. To understand this, let's return to the time evolution of the wave function. This is a complex valued object whose absolute value squared encodes the probability density of physical events. Clearly for this to make sense, the sum of the probabilities of all possible events must be unity, now and forever. The time evolution of the wave function is described by a linear operator (the Schrödinger equation), and our request that probabilities sum to unity at any time translates into the requirement that this operator be unitary. However, this rules out the behavior we concluded is exhibited by black holes. Let us imagine that, at some point in time, we would like to assure ourselves that all processes obey the law of unitarity. Clearly, we should tally up the probabilities of all possible experiments and discover that they add up to one. However, in the presence of an event horizon, some events become unknowable to the outside observer. The best he can do is assign conditional probabilities assuming complete ignorance about what happens behind the horizon. Worse still, the horizon eventually disappears and leaves behind nothing but perfectly thermal radiation, so that the information about these behind-the-horizon events can never be recovered. We can thus no longer preserve the unitarity of evolution, seemingly leaving us with nothing but bad choices concerning our fundamental theories.

2. We now refer to this temperature as the Hawking temperature of a black hole, in honor of Hawking's contribution.

3. Astrophysical black holes are in no hurry to evaporate: a solar mass black hole will take a cool 10^{67} years to disappear. Smaller black holes, on the other hand, disappear almost immediately, a Planck mass black hole vanishing in a time of the order 10^{-40}s. The smaller they are the hotter they get.

4. In the most general case, there will be angular momentum and charge in addition, but no more. The fact that a black hole, as well as its Hawking radiation, are fully described by three numbers is referred to as the "no-hair theorem".

In search of lost information

This then is the paradox of information loss: putting all our trust in gravity we find that quantum mechanical unitarity cannot survive as a principle of nature. Conversely, putting all our trust in unitary quantum mechanics forces us to conclude that the gravitational description of event horizons must be fundamentally flawed. The big question is then, "what gives?".

Major strides have been made to give a more solid underpinning to the very idea that black holes are thermodynamic objects. For an ordinary gas, like the air around us, thermodynamic properties arise as a statistical description of a myriad of microstates, in this case motional and internal degrees of freedom of the molecules. For a black hole, this question is a great deal more difficult to answer, for it essentially poses the question "what are the molecules of space time?". Within the context of string theory, such a microscopic account was first given by Andrew Strominger and Cumrun Vafa, who demonstrated that in some situations the quantum states of a conformal field theory give rise to the required microstates of so-called supersymmetric black holes. Nowadays we understand this calculation in terms of a broader framework, namely that of holographic duality, a major thread of current research. This is an approach to quantum gravity that takes the area scaling of black hole entropy as its starting point and consequently proposes to describe all of the physics of a given volume of space time by a theory that lives on its boundary. The holographic boundary description is a perfectly ordinary, and thus unitary, quantum field theory. This point of view, however, immediately poses the inverse challenge: how is it possible that non-unitary behavior of black holes can arise out of a unitary boundary description? It turns out that this question is closely tied to the problem of describing the black hole interior by using only quantities that make reference to the boundary theory and its observables. This, however, appears to be very complicated, even conceptually, which one can appreciate by recalling that the region behind the horizon is shielded from contact with the boundary, by the very definition of a black hole.

It has even been proposed, that the black hole interior does not actually exist, replacing all quantum states behind the horizon by a mere rearrangement of everything outside without adding fundamentally distinct degrees of freedom. Yet other proposals question the existence of the interior because their proponents believe that space time itself comes to a violent end at the event horizon in and attempt to avoid potential contradictions within well-established theorems in quantum information theory.

All of these variations on the theme of information loss (and others beside them) have their hotly debated merits as well as drawbacks. Arguably, what is most needed is a solid understanding of the very principles of quantum gravity, and importantly, a computational framework to put them into practice. Recent progress in the detection of black-hole mergers notwithstanding, this is all the more pertinent, given that the physical effects surrounding the paradox take place on scales far removed from our present experimental capabilities. It is remarkable that even now, 41 years after it was first posed, the black-hole information loss paradox is still setting the agenda in quantum gravity research and it will be exciting to see whether the current renewed impetus will finally carry fruit, allowing us to resolve the paradox one way or another.

Nothing is impossible, or more precisely, very improbable

Author: Julian Sonner
Associate Professor, UNIGE

Conversation with Luis Alvarez-Gaume

Luis Álvarez-Gaumé is a Spanish theoretical physicist who works on string theory and quantum gravity. He received his Ph.D. from Stony Brook University in 1981. After positions at Harvard and Boston University he joined the Theory Division at CERN as a Senior member. He was the Deputy Head and the Head of the Theory Group for a number of years. His research interests include: Quantum Field Theory, String Theory, Cosmology and Particle Physics.
As of September 2016, Dr. Luis Álvarez-Gaumé took the position of Director at the Simons Center for Geometry and Physics.

- What is your experience of working at the borderline between physics and mathematics? How, in your view, do the two subjects stand in relation to one another?

Well, good question. I was at CERN for many years, where I was working more in the field of physics, given it's an accelerator after all. Now, I'm the director of the Simons Center, which focuses on geometry and physics, so I interact every day with both physicists and mathematicians, and I feel quite comfortable in this particular "no-man's land". I have a fairly good background in mathematics, so I can more or less understand their language, but I feel more comfortable with physicists because I have my own habits after being around physicists most of my professional life.

It's very interesting to be in a place where you have both mathematicians and physicists, thinking about similar problems, because we look at the same objects but we ask different questions. In fact we have different ways of getting results. Mathematics is something that has to stay forever, so the timing and tempo are completely different from physics. Once you prove a theorem, if there is no mistake, it is there for eternity. Now in physics, you may have an extraordinary elegant theory, but if nature does not care about it, then it will be forgotten immediately. Sometimes, the notion of how well the proof is made, how well it is presented and so on, is not as pressing in physics as it is in mathematics. In mathematics you are talking about eternity, whereas in physics it is not so obvious. Perhaps most of the time eternity will evade you. Something like this creates, from a philosophical or even practical point of view, a huge difference: physicists sometimes are not so picky about details, because at the end of the day (in an ideal world), what counts is falsifiability which is not based on logic, but on experiment. Once you understand these differences, then you can survive very well in both communities, because you know what motivations and priorities each side has.

- Which directions would you recommend to a young graduate student interested both in physics and mathematics?

Well I prefer not to make recommendations. First of all, if the students are good enough, they do not need my guidance. All they have to do is keep their eyes and ears open, read and listen; and then if they have enough time (and perhaps some food) they can actually think independently and this is probably best.

I do think we still have plenty of interesting open problems. In particular in gauge theories, the relation between gauge theories and gravity, and holography, where both from a physics and mathematics point of view, there are still largely unexplored areas. These are areas that allow you to study, and learn a number of tools and subjects which are very interesting.

So I would suggest they look at a few areas, where they can learn many things that could be useful even if they eventually do condensed matter physics, or something else. Sometimes, graduate students have one computation to do and they're happy. They go to their room and they compute details but once they get through that particular barrier and show that they can solve problems, it's good to give them a challenge. Because they don't have to publish and they don't need to apply for jobs immediately, they can finally free their minds and spend one year hitting their heads against the wall, trying to explain something interesting. That's my philosophy.

- The ideas of string theory go back quite a number of years. How do you see the current state of your field?

That's a very interesting question. String theory is almost 50 years old. The first paper was Gabriele Veneziano's in 1968, the famous "Veneziano amplitude". So next year will be the 50th anniversary of the Veneziano formula, which eventually started the whole thing. Now, the problem with string theory, in many respects, is that it's like a tailor for all kinds of suits. String theory is a collection of tools and ideas which is very interesting and very deep. It's a mix between mathematics and physics, but the Popperian notion of falsifiability, has not been fulfilled. In string theory for example, you can make very interesting statements about black holes, properties of black holes, their entropy counts and all these other things, but unfortunately, these are things we cannot observe. And of the things we can observe there's nothing, or very little, we can say. So in a sense, maybe it is still too early. There are many things we have not yet learned about the deep structure of string theory, and we still have to wait until we find the right way of understanding whether it can be falsified like any other physical theory, or not. Perhaps we will have to change the notion of what a scientific theory is, or change the notion of falsifiability when nature is involved.

> **It's very interesting to be in a place where you have both mathematicians and physicists, because we look at the same objects but we ask different questions.**

The aspect in which string theory is extremely successful is in inspiring mathematics. This is what you might call experimental mathematics. Mathematicians don't care for example, whether Calabi-Yau manifolds are fundamental in counting the number of leptons or the number of quarks. For them what is important is that physicists found, through the use of physics and mathematics, mirror symmetry. Mathematicians, probably, could never have thought of it in their lifetime, because it is something so deeply rooted in the way we think in physics. Mathematicians think about a Calabi-Yau manifold as something very concrete, a collection of triangles, cubes and so on, but in physics, when you describe them in terms of string theory, it becomes an operator algebra, and then there is much more flexibility in how to deal with them. That's why physicists discovered mirror symmetry. And that's something that mathematicians are very happy about, because many of them got inspired by this. Many of them got important projects justified by studying this particular symmetry. So in that sense, string theory is clearly an incredible source of inspiration for mathematics and is ideal for interaction between both fields. That is also why there is an incredibly good connection between physicists and mathematicians in string theory related matters.

But more traditional physicists would say that string theory is not physics. They say, jokingly, that string theory is the future of physics… and it will always be. And this somehow means that it is not physics. I do think that one has to have a critical attitude, because at some point, we cannot become like monks in ivory towers, discussing things that cannot be tested. And since all is only based on logic, you have to be initiated to belong to the sect. So it becomes like a sect of physics, and that's pretty bad and could be very dangerous. I doubt it will happen in string theory.

- At CERN, you were a theorist in one of the world's biggest experimental labs. Can we hope that CERN might find some experimental evidence of string theory?

I think it is very unlikely. There are only two ways in which CERN could find evidence towards string theory. The first, which is the one that is nearly forbidden to speak about, is the formation of mini black holes. So far experimentally it seems to be clear that we are never going to produce these types of monsters at the LHC. That's a pity, because by looking at micro-states of black holes we could, in principle, understand many predictions of string theory. For example people talk about entropy which is the logarithm of a number of states. The number of states is an integer, which is very big, but in string theory you can compute that number exactly. So in a sense, it gives you an infinite number of predictions as a function of the mass of the corresponding black hole. So if you have a microscopic black hole, say at 50 TeV, then you have an extraordinary delicate prediction about the properties of the particular object. That would be a magnificent verification of string theory. But that's not the case, we haven't found them, and we haven't seen them, so we cannot experiment with them and test our theories.

The second is supersymmetry. People got carried away with the fact that string theory implies low energy supersymmetry. The difficulty is that from the point of view of string theory, low energy supersymmetry, could be the LHC energy or a thousand times the LHC energy. In fact, with all the landscape of structures, somehow it seems that almost anything fits inside string theory. And that's very bad news. On the other hand we could discover, the analogue of a gluino – a colour particle, the supersymmetric partner of the gluon but with spin ½, or squarks – particles like quarks but behaving like bosons. That would be the smoking gun for supersymmetry, and we'll all be celebrating for years. Because once you discover supersymmetry, then supergravity is next door, and that, more or less, almost inevitably leads to string theory. But who knows, maybe we are too proud intellectually and nature may actually have quite a number of surprises ahead of us. I do think that we are in for many surprises, and we should be humble and listen to nature. In the past for example, great physicists like Richard Feynman, would say that, when in your subject *you are stuck in a particular program, there are many other interesting areas in physics where you can make an impact or a contribution. These days, people are more mono-thematic. It is like people saying "I was born a string theorist, and I will die a string theorist". This is ridiculous. I think people should have a slightly broader view of the world and of physics. If you don't find a way to crack a particular problem, eventually, one will find the tools or the experimental information that will open up the problem. So I think that the LHC unfortunately, will not test string theory.*

- What is your view of SwissMAP and its activities? What aspects are particularly favourable for fostering developments in mathematical physics?

I think the most interesting part of these types of initiatives, is that we get together. There are small groups in different parts of Switzerland, including CERN, doing different types of activities, and I think it's very refreshing that we talk to each other, and criticise each other. Also important, is the fact that we can share young people and students. I think it's good for the students to go from being in a small group with perhaps a narrower scope, to being able to circulate within different groups. These groups are actually top centres too, like ETHZ, EPFL, CERN and Geneva, but none of these groups have critical mass. So by uniting forces, we can be much more competitive, our students can get better trained, our young postdocs can get more visibility and better jobs, and again the realm of projects that we can attack is broader. So I think that this is a great initiative, but of course for this we need funds, otherwise, you cannot attract students and young people.

There are small groups in different parts of Switzerland doing different types of activities, and I think it's very refreshing that we talk to each other, and criticise each other.

So far the initiative has been really good. The fact that people can discover different communities, will be a great benefit for their futures. So I hope that SwissMAP gets renewed for the 2nd phase!

- What were your first plans in the capacity of the director of the Simons Center? What did you implement first?

Well usually, when you come to a new place, you first look for the skeletons in the closet. And when you stop looking in the closet, you start digging in the garden to see how many skeletons are outside! Once you've found them, you try to understand if there is a future and how long a program can last. If you don't have enough financial stability it's very difficult to think that people will be attracted to this particular place. So one of the main things that I've been doing, which is pretty boring, but absolutely necessary, is to put in place in some kind of sound financial basis, so we can operate for say, the next five to ten years. To my surprise this was not obvious when I arrived. So I spent more or less the last 8 – 9 months trying to solve this particular problem.

Once you have this problem more or less solved, then you also have to plan out the number of jobs that you can offer. For example, I want to attract more junior faculty and students to the centre. Currently, we have very senior faculty and very junior people, who are first year postdocs, but we lack students that are financed by us, and we lack junior faculty. I think it would be great to share the junior faculty with the University, because then you can really compete with major institutions. If you want a place to become a reference point in say 50 years, you have to make sure that you not only attract people who are great, but who did things somewhere else. You want to make sure that young people will explode here. They will notice us, not because we have great facilities and we run great programs, but because we do absolutely great research. So the goal is to put down the initial conditions for this to happen. Now if it happens next year that will be great but it doesn't have to happen so fast. What's important is that it eventually happens. It is still a very young centre and there are many things that have been happening, so it is now about time to start working in that direction.

- Do you see any perspectives of collaboration between the Simons Center and SwissMAP?

Sure, plenty. We run all kinds of workshops, and programs, and there is no reason why it should only be localised here. We could share people like postdocs, junior faculty or students. Or we could have some workshops and programs that last from one week to say, six months, between the various institutions in SwissMAP and the Simons Center.

It's just a question of sitting down, planning it and doing it. And for us it's very simple. You want to do it, we agree on something and in a few months, it's done.

Conversation with
Luis Alvarez Gaume
May 2017, Geneva/Stony Brook

Interviewed by Maria Kondratieva
and Paul Turner
NCCR SwissMAP

Presenting the SwissMAP

Master Class 2016-17

in Geometry, Topology and Physics

The 2016-2017 Master Class covers some of the most important and actively developed subjects of the research area in-between geometry, topology and physics providing an entry point into the forefront research for students starting to work in this field.

Geometry and physics have been interconnected since ancient times, providing inspiration and intuition, as well language for each other. Geometric ideas lay in the foundations of electromagnetism, special and general relativity, while symplectic geometry is the modern language of mechanics, which, in turn, provides it with methods and motivation.

The program is aimed at Master students and beginning PhD students. The participants will enrol in a one-year program at the University of Geneva starting in September 2016, providing 60 ECTS credits. Participants will be offered the possibility of obtaining a master degree from the University of Geneva by completing a Master thesis for 30 additional ECTS credits.

The program started in September 2016 and will be completed in June 2017. 18 students from 10 different countries are enrolled in this Master Class alongside local students who were invited to attend the lectures.

On the following pages you will find an introduction to the lecturers and the chosen students.

The Lecturers & Courses

Anton Alekseev

Symplectic geometry of moment maps & Field theory for mathematicians

Prof. Alekseev is a full professor at the university of Geneva. His research interests include: symplectic geometry, moment theory and mathematical physics.

Sergey Galkin

Symmetries and moduli spaces I

Prof. Galkin teaches mathematics at the Higher School of Economics and Independent University of Moscow. His research focuses on algebraic geometry.

Ilia Itenberg

Topological aspects of algebraic geometry I

Prof. Itenberg is a full professor at the IMJ-PRG. His main fields of interest are real algebraic geometry, topology of algebraic varieties, symplectic geometry, tropical geometry and enumerative geometry.

Rinat Kashaev

Introduction to quantum topology II

Prof. Kashaev is an associate professor at the university of Geneva. His research interests are knots theory, hyperbolic geometry and mathematical physics.

Marcos Marino

Quantum mechanics for mathematicians

Prof. Mariño is a full professor at the university of Geneva. His research interests are mathematical aspects of string theory and quantum field theory, topological string theory, topological quantum field theory, quantum gravity and non-perturbative methods in quantum field theory.

Grigory Mikhalkin

Topological aspects of algebraic geometry II

Prof. Mikhalkin is a full professor at the university of Geneva. His main fields of interest are topology, algebraic geometry and symplectic geometry and connections to physics and other areas.

Pavol Severa

Poisson geometry and quantization

Pavol Severa is a senior lecturer at the university of Geneva. His research interests include Poisson geometry and mathematical physics.

Alexis Virelizier

Introduction to quantum topology I

Prof. Virelizier is a full professor at the university of Lille. His main fields of interest are quantum topology and it's associated algebraic structures.

Full list of courses:

First semester courses:

1. Quantum mechanics for mathematicians (*M. Mariño*)

2. Topological aspects of algebraic geometry I (*I. Itenberg*)

3. Symmetries and moduli spaces I (*S.Galkin*)

4. Symplectic geometry of moment maps (*A. Alekseev*)

5. Introduction to quantum topology I (*A. Virelizier*)

Second semester courses:

6. Introduction to quantum topology II (*R. Kashaev*)

7. Topological aspects of algebraic geometry II (*G. Mikhalkin*)

8. Field theory for mathematicians (*A. Alekseev*)

9. Poisson geometry and quantization (*P. Ševera*)

10. Winter School in Les Diablerets with talks by: Anton Alekseev, Jürg Fröhlich, Geoffrey Scott and Yuri Tschinkel.

All classes held in UNIGE can be viewed on the MediaServer: https://mediaserver.unige.ch/collection/VN3-222c-2016-2017

Willem (Rik) Voorhaar, Arina Arkhipova and Elizaveta Arzhakova.

The second SwissMAP Master Class 2016-17 welcomed 18 international students, hand-picked by our committee based on their academic skills, knowledge and achievements.

The students come from a large variety of countries: Belgium, Canada, Colombia, Iran, Italy, Japan, the Netherlands, Russia, Switzerland and the United States.

We decided to ask some of the students a few questions about this year's Master Class:
Elizaveta Arzhakova (Russia);
Benjamin Hoffman (USA);
Arina Arkhipova (Russia);
Willem (Rik) Voorhaar (The Netherlands);

Q: What part of this year's Master Class did you like the most ?

Elizaveta: I liked the opportunity to attend interesting lectures, while having free afternoons to do some research activity. I like that we are granted an opportunity to pursue projects with the members of the faculty. The great thing is that all lectures are filmed. I find it great that at the end of the year the lecturers show us the intertwining of all the subjects that we had during both semesters. I also love the city and I cannot get enough of seeing the Alps every day!

Benjamin: I enjoyed seeing multiple perspectives on mathematical physics, and working with an international community of mathematicians.

Arina: I really enjoyed the format of this event. The idea of selecting a small group of people and teach them some advanced courses is not new, but such events usually do not last longer than a month or two, therefore, the participants do not have enough time to get acquainted with the subject. Thus, a one-year program is definitely a much better way to learn something new, to

Lyalya Guseva.

work with famous researchers and even conduct some research under their supervision.
Also, I can't not mention that I really enjoyed Ilia Itenberg's semester course on Topological Aspects of Algebraic Geometry. In my point of view, Ilia is one of the best lecturers I have ever met in my mathematical life, and I am really grateful to the organizers of the Master Class for inviting him to give us this amazing course.

Rik: Most of the courses taught are very interesting, and they are given at a higher level than most courses available to me in the Netherlands.
I think it's great to be able to work together on advanced mathematics with people with similar interests.
The Master Class also has a very nice informal attitude; we're not required to do anything in particular and exams are completely optional. While I'm still doing most exams, the informal attitude here really reduces stress and allows me to focus on doing the mathematics that interest me instead of worrying about course requirements.

Q: What will you be taking away from this experience?

Daniel Chupin.

Elizaveta: I am very lucky to have been selected for the Master Class 2016-2017. During this year, I gained diverse knowledge on topics from physics, topology, and geometry. This certainly broadened my mathematical knowledge, and I will profit from it in my further academic career. I am also happy to have a research project with Anton Alekseev, through which I learned much on probability theory. This project teaches me new approaches to problem-solving, and it is a valuable research experience.
Thanks to this Master Class, I made great friends with my groupmates. They make me enjoy every day of my stay in Geneva, they support me and teach me how to be a better person.

Benjamin: The most valuable thing for me has been my collaboration with Anton Alekseev and Yanpeng Li. I've learned a lot working on the project, and I'm looking forward to continuing our work in the future.

Arina: I suppose, I am not the only person that will say that this Master Class has made my life better. I have met lots of nice people, attended lots of interesting talks, read several books and obtained some experience in various areas of mathematics and mathematical physics. And the greatest thing is that the Master Class gave me the supervisor for my PhD studies!

Rik: After (almost) a year of studying here in Geneva I feel like I'm much closer to research level mathematics than I was last year, and I have a much better sense of what mathematics interest me the most. Before coming here I only had vague notions of what kind of mathematics I wanted to do, but now I have a fairly good idea of what I want to study next year when I go back to Utrecht to write my master thesis.

Q: What improvements does this program need?

Elizaveta: In my opinion, the Master Class would benefit from homework and obligatory exams.

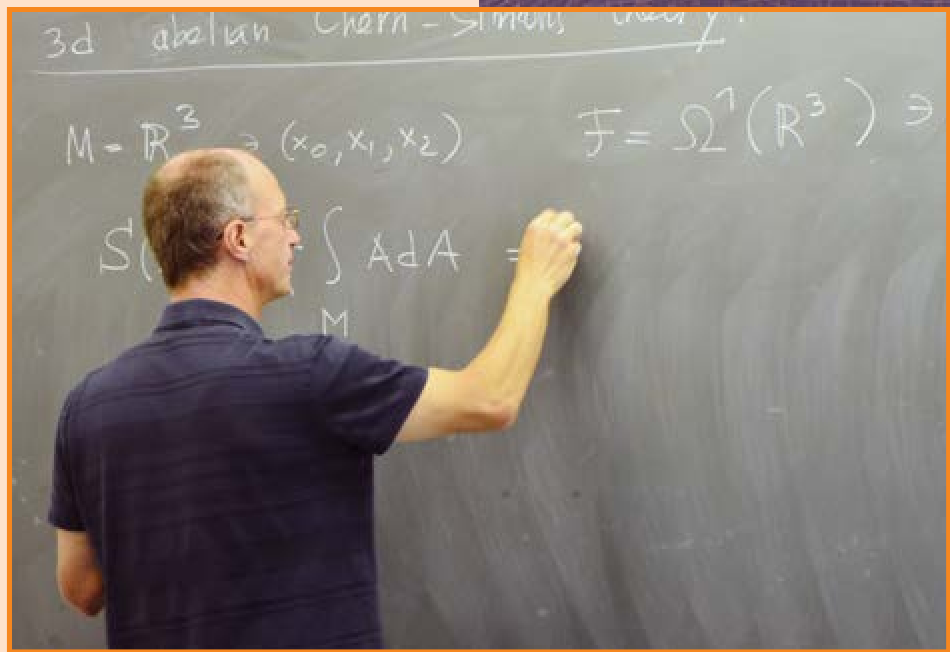
Benjamin: There was a bit of overlap in classes, though with different notation and perspective. It would have been nice to see things presented in a more unified way. For instance, we saw constructions related to braided and symmetric monoidal categories several times in the first and second semester, in Quantum Topology I and II, and Poisson Geometry. However, only in Poisson Geometry were the details really spelled out.
Also, it would be useful to have had some specific papers to look at for the more research focused classes, for instance in quantum topology.

Arina: It would probably not be a bad idea, if in the next Master Class,

Celia Hacker

the research segment was more developed, providing an opportunity for students to work together on some concrete problems, maybe in collaboration with local PhD students.

Rik: This is a hard question. I can't really think of anything, the Master Class is really good and doesn't need any improvement as it is.



Prof. Anton Alekseev during class.

All photos by: Maria Kondratieva

Events 2017

Past Events: Summer 2016
Over the 2016 summer break, SwissMAP held a high number of events.
The 2nd SwissMAP Site Visit was held at the ETHZ in Zurich. In June, SwissMAP held the Poisson 2016 conference and summer school which was co-organised between the Geneva and Zurich offices. The Integrable Systems conference was held at the CSF in Ascona. In August, SwissMAP co-organised the Raqis'16 conference - a satellite conference of STATPHYS 26.

Jan – Feb

09 – 13 January
Winter School in Mathematical Physics
Les Diablerets
The annual Winter School in Mathematical Physics will take place in Les Diablerets from January 09 to January 13, 2017.

16 – 19 January
Informal Geometry/Topology workshop
Belalp
The Informal Geometry/Topology workshop will be held at the Belalp in Valais on 16 - 19 January 2017.

22 – 27 January
Integrable random systems, representation theory and geometry of Lie groups
Les Diablerets
This workshop will take place in Les Diablerets on January 22-27, 2017.

12 – 18 February
Workshop in Statistical mechanics
Les Diablerets
The Workshop in Statistical mechanics will take place in Les Diablerets and is organised by Stanislav Smirnov.

Mar – Apr

22 – 24 March
In memory of Ludwig Faddeev
UNIGE, Geneva
This series of talks will present some of Faddeev's major contributions at the edge between mathematics and physics.

May – June

01 – 02 June
3rd SwissMAP Site Visit
UNIGE, Geneva
The 3rd SwissMAP Site Visit will be held at the University of Geneva and will include a Poster Session by our students.

07 – 09 June
Swiss Knots 2017
University of Bern
Swiss Knots 2017 is part of a series of conferences on knot theory and low-dimensional topology organised in Switzerland.

12 – 16 June
23rd Rolf Nevanlinna Colloquium
ETH Zurich
The colloquium will take place in the ETH main building.

16 – 18 June
Women in Geometry and Topology
ETH Zurich
SwissMAP is organizing the conference "Women in Geometry and Topology" that will bring together leading experts and young researchers.

July – Aug

27 Aug – 01 Sep
Lie groups in mathematics and physics
Les Diablerets
This conference will bring together specialists working on theoretical aspects of Lie theory and on its applications to physical models.

Sep – Oct

10 – 13 September
SwissMAP General Meeting
Grindelwald
The annual SwissMAP General Meeting will be held in Grindelwald this year.

For the complete list, please visit: http://nccr-swissmap.ch/events

Awards

Hugo Duminil-Copin

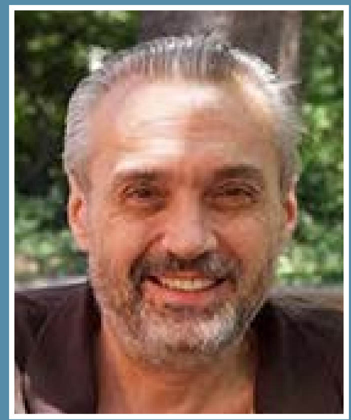
New Horizons Prize

Congratulations to Hugo Duminil-Copin who was awarded the New Horizons Prize for brilliant solutions to multiple landmark problems in probability, particularly regarding critical phenomena for Ising-type models.
Prof. Duminil-Copin is a permanent professor at the Institut des Hautes Études Scientifiques (IHÉS) and professor of mathematics at the University of Geneva. He received the IAMP Early Career Award in 2015. His research deals with mathematical physics, probability, complex analysis and combinatorics.

Tudor Ratiu

Tullio Levi-Civita prize

Congratulations to Tudor Ratiu for receiving the international prize Tullio Levi-Civita for the Mathematical and Mechanical Sciences.
Tudor Ratiu is a professor emeritus and Head of the Chair of Geometric Analysis in the mathematics section of the Ecole Polytechnique Federale de Lausanne (Swiss Federal Institute of Technology Lausanne). He also founded the Bernoulli Center, a mathematics research institute. Most of his research centers on geometric mechanics (both classical and continuum) and nonlinear global analysis.



Wendelin Werner

Heinz Gumin Award for Mathematics

Congratulations to Prof. Wendelin Werner for receiving the Heinz Gumin Award for Mathematics of the Carl Friedrich von Siemens Foundation, for his ground-breaking contributions to the mathematical understanding of universal properties of Brownian motion and applications on central assumptions of statistical physics.
Prof. Werner is a professor of mathematics at the ETHZ in Zurich. He was awarded the Fields Medal in 2006. His research focuses on random processes such as self-avoiding random walks, Brownian motion and Schramm-Loewner evolution.

Michele Maggiore & Andreas Mueller

Credit Suisse Award for Best Teaching 2016

Congratulations to Prof. Michele Maggiore and Prof. Andreas Müller for receiving the "Credit Suisse Award for Best Teaching 2016 - UniGe" for their creation of the Athéna program.
Both are professors at the University of Geneva. Prof. Maggiore's recent research is largely devoted to non-local modifications of GR and Prof. Müller is the head of the Didactic of Physics research group.

Grants

Clement Hongler

ERC Starting Grant

Clément Hongler has received an ERC Starting Grant titled "Connecting Statistical Mechanics and Conformal Field Theory: an Ising Model Perspective".
The goal of the project is to develop mathematically the connections between Statistical Mechanics and Quantum Field Theory, starting from the conjectured correspondence between the Ising Model and Conformal Field Theory. The goal is to make this correspondence completely rigorous, and then to expand it to other models and field theories, including in particular the Unitary Minimal Models and Integrable Massive Field Theories.

Antti Knowles

ERC Starting Grant

Antti Knowles has received an ERC Starting Grant titled "Spectral Statistics of Structured Random Matrices".

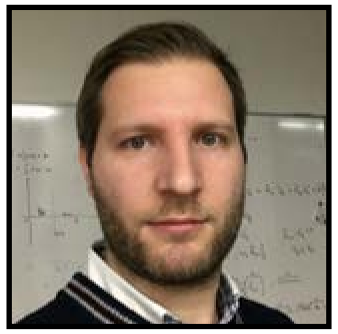
The goal of this project is to study the eigenvalue distribution of random matrix models arising from two areas: spectral graph theory and the physics of disordered quantum systems. Both types of model contain some nontrivial structure, which makes them at the same time more interesting but also more challenging to analyse than the Wigner-type mean-field matrices that have up to now been the main focus of random matrix theory.

New Collaborators

Vincent Tassion

ETH Zurich

Vincent Tassion obtained his PhD in 2014 at the ENS Lyon. He finished his post-doctoral studies in 2016 at the University of Geneva under the supervision of Hugo Duminil-Copin. He was appointed assistant professor at ETHZ as of September 2017. His research interests are in the mathematical study of questions arising in statistical mechanics, and more precisely in percolation theory.



Alessandro Vichi

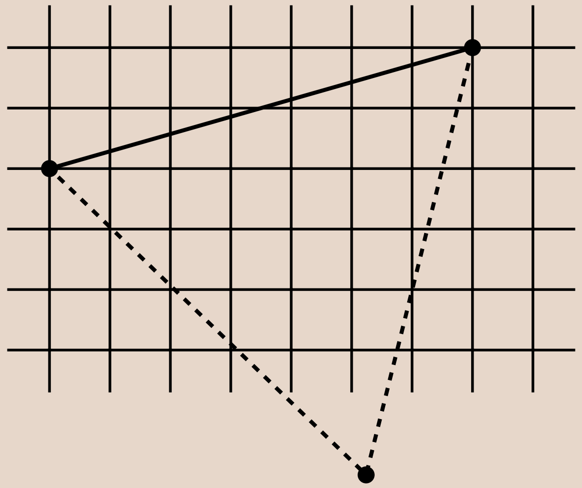
EPF Lausanne

Alessandro Vichi obtained his PhD at the EPFL, under the supervision of Professor Rattazzi.
After his PhD, he held a shared postdoctoral position at UC Berkeley and Lawrence Berkeley National Laboratory for three years. He was then a research fellow at CERN from 2014 to 2016, when he started a professorship at EPFL funded by the Swiss National Foundation of Science (SNSF).
He is teaching classes in advanced quantum field theory and quantum physics.

Puzzle Corner

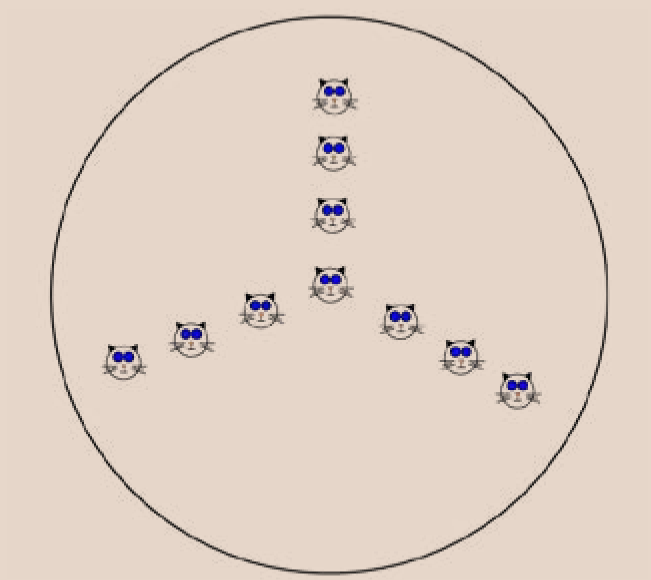
1. Vertices and Lattice Points

Is it possible to find an equilateral triangle, all of whose vertices are among the lattice points on an infinite square grid?



2. Separating Cats

You have to separate all the cats from each other, using only three perfect circles (the circles may intersect the big one)

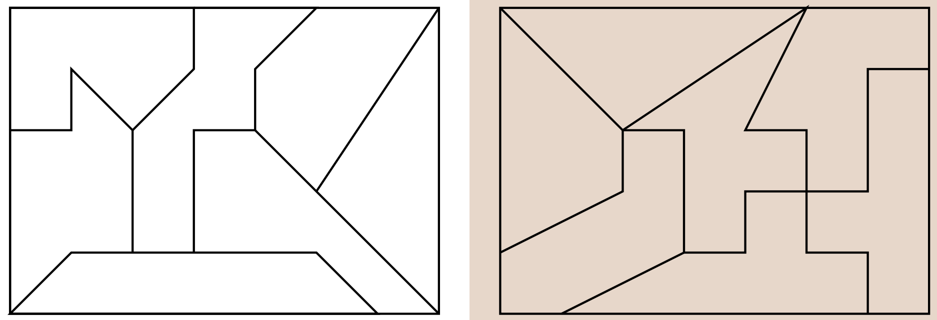


3. The Magician

Magician David Blaine and his assistant perform the following trick. There is a circle drawn on a blackboard. A member of the audience marks 2017 different points on the circle, then the assistant erases one of them. After that, David Blaine enters the room for the first time, looks at the drawing and shows a semicircle that contains the erased point. How can they agree on a strategy for this trick?

4. Two Rectangles

Two identical rectangles have been independently subdivided, each into N polygons of the same area. The two rectangles are then placed one on top of the other. Is it always possible to pierce them by N pins, so that each of the $2N$ polygons in the two subdivisions gets pierced exactly once? (No pin should hit the boundary of any of the polygons.)



5. The Kings Kingdom

A Sunday evening, a king has imprisoned two of the finest logicians in a tower. Each logician sees his half of the kingdom from the window of the tower. They are in separate rooms, and cannot hear or see each other. The king, playful, visits the logicians separately and offers them a way to escape: « There are 10 or 13 villages in my kingdom. You have to guess which of the two. Every day, a guard will come to your cell to ask you for the answer. If you answer correctly, both of you will immediately be set free; if you answer falsely, both of you will be killed that day; if you don't answer, you'll just stay in your cell. » The logicians both accept the deal.
Neither answers on Monday, Tuesday, Wednesday or Thursday. On Friday, they both answer correctly, and are set free.
How many villages were there? How were they distributed on the landscape?

6. Equidistant Coins

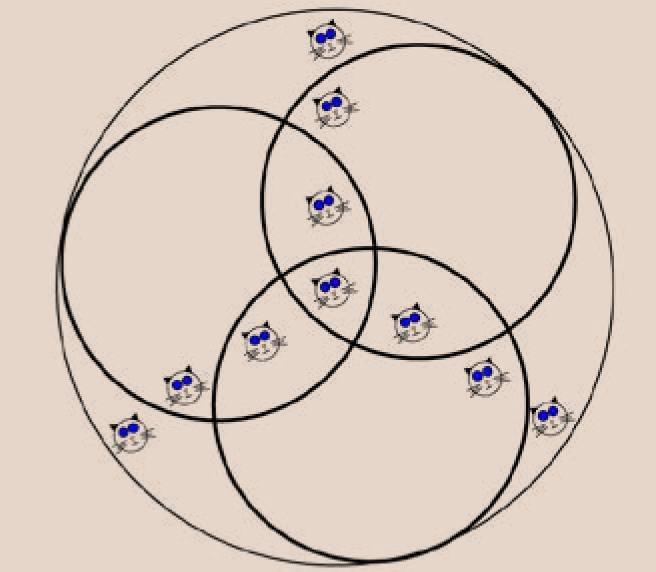
Would you be able to place a certain number of coins so that they are two by two equidistant? In other words, in such a way that the distance between two coins, whatever they may be is always the same?
For example, with three coins, you need to arrange them in an equilateral triangle.
Challenge: How can you place five coins so that they are two by two equidistant?

Answers

1. Vertices and Lattice Points

No, it is not possible.
Say the sides of the squares forming the grid have length 1. Then the area of any triangle with vertices among the lattice points is a *half-integer*. This follows either by expressing the area explicitly in terms of the coordinates of the vertices, or by Pick's formula, expressing the area as I + B/2 - 1, where I and B denote the number of interior and boundary lattice points, respectively. If an equilateral triangle of side length a has its vertices among the lattice points, its area $a^2\sqrt{3}/4$ would be a half-integer, hence, in particular, rational. However, a^2 is an integer, by the Pythagorean theorem, and so $a^2\sqrt{3}/4$ cannot be rational.

2. Separating Cats



3. The Magician

Here is one of the possible solutions. Consider 2017 arcs between the points. Let AB be the longest among them (if there is more than one, choose it arbitrarily), assume AB goes clockwise. Then the assistant will erase A. Now, when David Blaine enters the room, there is exactly one longest arc with the right endpoint B. Let C be the left endpoint. Then David Blaine can simply show the semicircle that starts at C and goes clockwise.

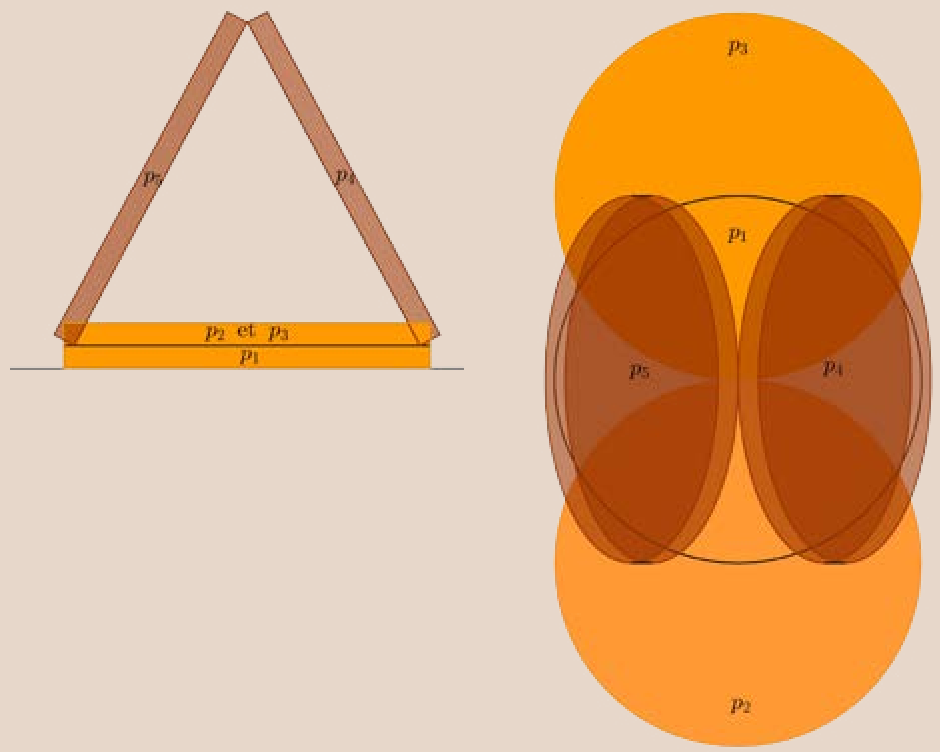
4. Two Rectangles

Yes, this is always possible. Let *P* and *Q* denote the sets of polygons in the subdivisions of the two rectangles. Construct a bipartite graph *G* as follows. The set of vertices of *G* is $P \cup Q$. Draw an edge between a vertex $p \in P$ and a vertex $q \in Q$ if there exists a pin that pierces both polygons *p* and *q* in their interiors.
Consider an arbitrary subset *X* of *P*. Since all *2N* polygons involved in the subdivisions have equal areas, there exist at least $|X|$ polygons in *Q* that can be pierced by a pin through a polygon in *X*. For the graph *G*, this means that the set
$\{y \in Q \mid$ there exists an edge between y and a vertex in $X\}$
has size at least $|X|$.
This is exactly the condition in Hall's Marriage Theorem, which guarantees a perfect matching in the graph *G*, i.e., a bijection $\sigma : P \rightarrow Q$ such that *p* and $\sigma(p)$ are connected by an edge, for every $p \in P$.

5. King and Kingdom

13 villages. 6 on one side, and 7 on the other.

6. Equidistant Coins



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Cover photo: Maria Kondratieva, *Les Diablerets*