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In theoretical physics, our best tool to calculate observable quantities is perturbation theory. We know very little about strongly coupled systems. Any new tool to access the strongly coupled regime is therefore most welcome. We found that working in sectors of large charge is way of compensating the effects of strong coupling.

I am a string theorist by training and in the course of my career I have gone through what resembles a random walk in the space of problems in formal theoretical physics. I started out in string phenomenology at a time when flux compactifications were a hot topic. I was most attracted by their most formal aspect, namely the algebraic geometry of the compactification manifolds. Motivated primarily by my interest in the (much more formal) field of topological string theory, I went to Amsterdam to work with Robbert Dijkgraaf for my first postdoc. While I never actively worked on this topic, it lead me to become interested in the connections between integrable systems and supersymmetric gauge theories. Realizing such deformed gauge theories appearing in these correspondence within string theory came next. This topic, and brane realizations of supersymmetric gauge theories in general kept me busy for quite a while through my moves to Japan and then to CERN.

To my own surprise, I ended up adding a quite different line of research to my collection. This happened in spring 2015, just after having started my new job at the University of Bern. I was in Japan, visiting my long-time collaborator Simeon Hellerman together with my collaborator (and husband) Domenico Orlando. For me, every new project tends to be a journey into the unknown, not just in the obvious sense that we don't know the end result of a research project when we start it, but also in the sense that it often involves physics I previously knew only little about. This topic was no exception. It is centered on the study of special sectors of three- and four-dimensional conformal field theories (CFTs). The most surprising part was that the problem had nothing to do with string theory or even supersymmetry (even though it is possible to consider superconformal field theories (SCFTs)).

Conformal field theories are, as the name suggests, invariant under conformal transformations. This gives rise to special features. If we know the operator dimension and spin of each local operator, conformal symmetry fixes the two-point functions up to normalization. If we furthermore know the 3-point function coefficients, we can solve the theory completely, in the sense that we can write down all higher correlation functions.

CFTs play an important role in theoretical physics, as they show up in a variety of contexts, such as critical points characterizing second order phase transitions, fixed points in renormalization group flows, and even quantum gravity via the AdS/CFT correspondence. CFTs are scale-free, meaning they contain no characteristic length or energy scale. This means also that we have no dimensionful small parameter in which to perform a perturbative expansion. The dimensionless couplings in a CFT are generically of order one. While in two dimensions the special nature of the conformal group allows us to use a host of analytic techniques, in higher-dimensional CFTs things are much more tricky. Of course, we have some methods at our disposal, such as large-N expansions, (small) epsilon expansions and the conformal bootstrap. And it is possible to run Monte-Carlo simulations on the lattice at strong coupling. But analytic results are still few and far between and any new approach that can help us get a handle on strongly coupled problems is important progress.

Susanne Reffert discussing large charge with Luis Alvarez-Gaume and Domenico Orlando at the Simons Center for Geometry and Physics. Photo by Jean-François Dars.

The approach we follow is quite a time-honored one in theoretical physics, namely making use of the symmetries of the problem and considering special subsectors of the full theory in which simplifications occur. In our case, we consider CFTs with a global symmetry. Such a symmetry has by Noether's theorem an associated conserved charge, which can be used to slice the Hilbert space of states of the theory into sectors labeled by their charge. We concentrate on a subsector of fixed charge, where we take the charge to be very large. It turns out that it is possible to write a low-energy effective theory in which the inverse of the large charge acts as a controlling parameter, bringing us back to a perturbative regime. Wilson's notion of the effective action in which any term compatible with the symmetries of the problem must appear is conceptually very compelling. If we don't have a way of truncating the effective theory, it is however of very limited practical use. Working at fixed charge allows us to do exactly that. In addition to using the constraints due to symmetry, we also use the fixed-charge scaling to discard those terms which are highly suppressed by inverse powers of the charge. In the problems we have studied so far, we were generally left with only a very small number of contributions which are not suppressed. In other words, working at large charge allows us to make the step from an effective action with infinitely many terms to an expansion which captures the low-energy physics in a handful of terms.

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We approach the problem semiclassically, solving the classical equations of motion at fixed charge and minimizing in order to find the lowest-energy state at fixed charge. This ground state has the special feature of being time-dependent. Working at fixed charge breaks both the global and the spacetime symmetries. Part of this breaking is explicit and due to fixing the charge, part of it is spontaneous and due to the ground state itself. Spontaneous symmetry breaking gives rise to massless Goldstone degrees of freedom. They represent the quantum fluctuations around the classical ground state and encode the low-energy physics in the effective action. If the global symmetry we started from was just a U(1), then things are simple and we are left with a single (relativistic) Goldstone boson in terms of which we write a non-linear sigma model. The action contains all the terms compatible with conformal symmetry which are not suppressed by the large charge. If we start from a larger, non-Abelian global symmetry group, we need to first determine the symmetry-breaking pattern. Since the ground state breaks Lorentz invariance, we are generally left with both relativistic and non-relativistic Goldstone bosons, which are distinguished by their dispersion relations (linear versus quadratic in the momentum). Once we have written down the effective action in the form of a large-charge expansion, we can start calculating the conformal data, namely operator dimensions and three-point coefficients, from which the general n-point functions can be determined. The energy of the ground state at fixed and large charge Q in particular gives via the state-operator correspondence of CFT directly the conformal dimension of the lowest-lying state of charge Q. The biggest contribution to the ground-state energy comes from the classical ground state, while the vacuum energy of the relativistic Goldstones gives a subleading contribution.

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One way of using the large-charge method is to apply it to known CFTs, such as for example the Wilson-Fisher fixed point in the infrared of the O(N) vector model in three dimensions. Another stance one could take is to simply assume that a certain CFT exists and apply the large-charge method to it. We have worked with models motivated from both condensed matter physics and particle physics. The large-charge expansion even works for non-relativistic systems with Schrödinger symmetry (as opposed to conformal symmetry). An example is the unitary Fermi gas which can be experimentally realized in the laboratory via cold atoms in a trap. Connecting back to more formal theory topics, the large-charge expansion can be applied also to superconformal field theories at large R-charge. Here, we found in particular that cases with a moduli space of vacua behave very differently from theories with one discrete vacuum I described above.

There is in general little known about the strongly-coupled models we are studying at large charge, so we have few results to compare our predictions to. But whenever there are results to compare to, be it on the lattice or from supersymmetric localization in the case of SCFTs, the confirmation of our predictions has been strikingly strong. When comparing with numerical results from lattice calculations, we found that our formulae derived at large charge even work down to very small values of the charge, which is highly unexpected (see Figure 1). The largest charge used on the lattice was 12, which is by no means a very large charge, but the agreement remains excellent even down to charge one. This has taken us very much by surprise, as it is far from the regime in which our effective theory is valid.
In the case of N=2 SQCD with 4 flavors, we could compare to a supersymmetric localization calculation which has again shown an amazing agreement and even allowed us to estimate the exponential corrections to the large charge expansion (see Figure 2).

Working on CFTs at large charge has been an extremely interesting and enriching experience for me, both scientifically and personally. On the one hand, this is the closest to "real-world" physics I have ever come, and I am really enjoying it. On the other hand, the problem has appealed to quite a varied group of people, getting me into contact with subfields I had known relatively little about before. Working on large charge not only got me talking to new people but has even lead to very interesting new collaborations outside my usual string theory community, such as the one with two lattice theorists, who recently verified our prediction for the O(4) vector model to high precision.

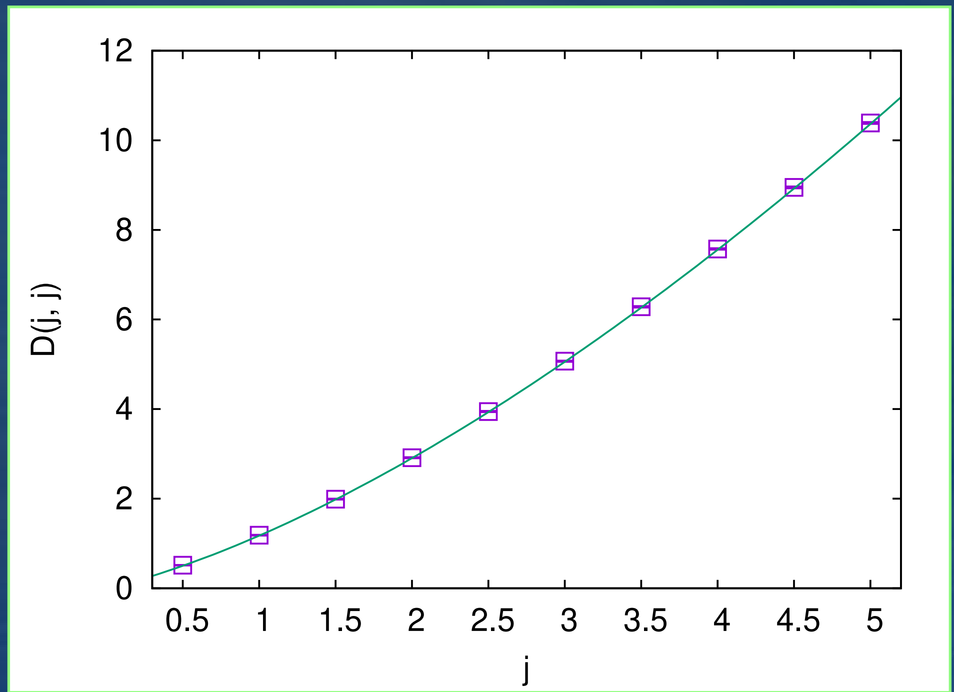

Figure 1 - Plot of the conformal dimension D(j, j) as a function of the charge j for the lowest operator of given charge in the O(4) vector model in three dimensions. The squares represent the data obtained using Monte-Carlo calculations on the lattice. The solid line is the large-charge prediction. Source: "Conformal dimensions in the large charge sectors at the O(4) Wilson-Fisher fixed point", D.Banerjee, Sh.Chandrasekharan, D.Orlando, S.Reffert, arXiv:1902.09542

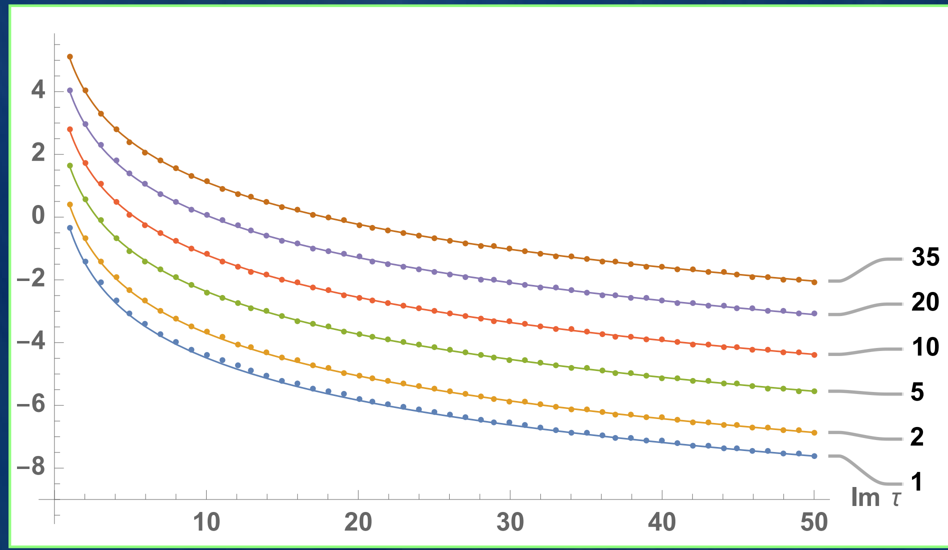

Figure 2 - Plot of the universal part of the 3-pt function coefficient of N=2 SQCD with 4 flavors as function of the gauge coupling for different values of the R-charge. The dotted lines represent the exact numerical results from the localization computation, the solid lines are our large-charge predictions, the numbers on the right side give the value of the fixed charge. Source: unpublished, based on results from "Universal correlation functions in rank 1 SCFTs", S.Hellerman, Sh.Maeda, D.Orlando, S. Reffert, M.Watanabe, arXiv:1804.01535

Since our original paper in 2015, Simeon Hellerman and his (now former) student Masataka Watanabe, Domenico and I have continued to push the large-charge expansion forward in varying configurations of new and old collaborators. But it's been especially great to see that the topic has attracted a number of independent groups around the globe. In late summer of this year, I am co-organizing (together with Domenico, Simeon and Luis Alvarez-Gaume) a 1-month workshop at the Simons Center for Geometry and Physics, where we are hoping to bring together a varied set of people interested in systems at large quantum number and related topics in order to explore new approaches, connections and applications.

I believe that we've only scratched the surface of the power of the large-charge expansion and that the method can be widely applied and developed in several new directions. Since we are in the unique position of having a good theoretical handle on strongly coupled systems, the large-charge approach might for example allow us to explicitly check conjectured (strong/weak) dualities.

When we first started working on large charge four years ago, at least I had no idea what I was getting into. It's been quite a ride and I've learned so much! The large-charge expansion has exceeded our boldest hopes when we started on the project. I am excited both to be part of and to watch its development and hope that it holds many more exciting surprises for us. Who knows where it will lead me next on my path of theoretical physics research?

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